Impact of monetary policy on corporate default and associated welfare cost

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Abstract

This study investigates the impact of modern monetary policy instruments, particularly quantitative easing and forward guidance, on corporate default and the resulting welfare costs. To this end, the study employs a competitive equilibrium framework that features cash-in-advance constraints and endogenous corporate default. We find that both monetary policies decrease the default risk, but quantitative easing escalates the welfare costs of corporate defaults, whereas forward guidance mitigates them. At the medium level of default, the estimated welfare cost of corporate default attributable to quantitative easing and forward guidance is 0.0488% and 0.0494% of real output, respectively. Our findings also reveal that including endogenous default in the competitive equilibrium model enhances sensitivity and responsiveness to monetary policy fluctuations, demonstrating that default risk functions as a pivotal connector between the real economic landscape and the financial sector.

Keywords: cash-in-advance economy, monetary policy, corporate default, welfare cost
JEL Classification: E37, E52

1 Introduction

"Monetary policy is not a panacea, but money matters, a great deal" (Bernanke, 2022). In the 21st century, the US Federal Reserve Board (Fed) and other major central banks worldwide have illustrated this statement by lowering financial frictions during economic and financial crises. Central banks used two monetary policy tools after conventional rate cuts were exhausted, including large-scale quantitative easing and explicit forward guidance[1], which were intended to influence financial conditions by shaping market expectations of future monetary policy (Bernanke, 2022). Though the effectiveness of these monetary policies on economic activity is witnessed in both academic research and practical experience, few studies have discussed their impact on default risk in the corporate sector and its consequential impacts on welfare costs in the household sector.

(Modeling money under a cash-in-advance model)

This study estimates variations in the corporate default rate and the cost of default on households ' welfare in response to monetary policies of quantitative easing and explicit forward guidance. We use cash-in-advance (CIA) models to model monetary policy mechanisms in a general equilibrium model. Lucas (1982) and Lucas and Stokey (1987) constructed CIA models in which goods can be identified as distinctive "credit" or "cash" goods depending on timing and agents hold money due to CIA constraints (Clower, 1967) for buying cash goods. A Clower type CIA constraint implies that agents must have enough cash balance before they can purchase cash goods, capturing the role of money as a medium of exchange. Following the basic framework of Lucas and Stokey (1987) and Hansen (1985), Cooley and Hansen (1989) introduced money into the real business cycle model, revealing the co-movement between real and nominal economic variables. CIA models then became a popular approach to model money in the dynamic macroeconomic framework because it offers a simple approach for modeling money flow and reflecting the effects of quantitative easing (Fuerst, 1994; Michel and Wigniolle, 2005; Chugh, 2009; Yanagihara and Lu, 2013; Chen, 2018).

(Modeling forward guidance)

We also focus on the second-order effect of money flows to describe explicit forward guidance. Our definition references previous findings that more volatile policies can cause more confusion concerning central banks' intentions, which leads to delayed investments (Fernández-Villaverde et al., 2011; Baker et al., 2012; Stokey, 2013; Bloom, 2013; Pastor and Veronesi, 2013). However, our study is the first attempt to examine the implications of explicit forward guidance on default as an equilibrium phenomenon by employing a monetary policy rule that incorporates time-varying volatility. Referencing Baker et al. (2012), we treat forward guidance as an exogenous stochastic process based on the forward-looking nature of policymaking.

(Default in the general equilibrium model)

Since default has a vital influence on financial fragility, some studies have included the possibility of default in general equilibrium models. This approach was first proposed by Shubik and Wilson (1977), in which agents in the model are allowed to choose repayment rates wherein the equilibrium model can accommodate partial or complete abrogation of agents' contractual obligations. When agents are not held accountable for repayment, the result can be predictable; they will rationally opt not to repay their debts. Following this setup, Goodhart et al. (2006) and Tsomocos (2003) modeled endogenous default into the benchmark economy to examine the role of default in the monetary mechanism. We adopt this concept of default into the CIA model, permitting corporate agents to

[1]Forward guidance is a tool that central banks use to communicate with the public about the anticipated direction of future monetary policy. When such guidance is issued, it informs the decision-making processes of individuals and businesses entities regarding their spending and investment choices. Consequently, announcements about prospective policies can have an immediate impact on current economic and financial situations. The Fed started using forward guidance as a monetary policy tool in the early 2000s.

forego debt repayment when it is deemed unworthy.

(Default penalty)

Penalties have been introduced in the literature to constrain corporate default such as pecuniary or nonpecuniary costs. Pecuniary penalties refer to garnishing future income or search cost for new financial resources, and nonpecuniary cost refer to criminal prosecution or loss of reputation. Shapley and Shubik (1977) introduced minimal institutions such as money, credit, and default and the rules of default penalties to be levied against agents who choose to default. Regarding nonpecuniary default penalty, Shubik (1973), Shubik and Wilson (1977), and Tsomocos (2003) incorporated default penalties on borrowers into a general equilibrium framework, introducing default cost that directly reduces borrowers' utility. Dubey et al. (2005) imposed a default penalty in the form of reputation loss in a multi-period economy that compels agents keep their promises to pay back debts, contending that firms fear losing new financing opportunities in the future from any partial default. Tsomocos (2003) and Dubey et al. (2005) modeled default penalties by subtracting a linear term from the default agents' objective function. In Dubey et al. (2005), the default penalty is proportional to the nominal size of the debt. This study follows the latter form; however, we introduce a default penalty that is in nonlinear form, where the assumption of linear default penalty disappears in the first-order condition (FOC).

(Welfare cost of default)

When companies opt to strategically default to maximize profit, this decision causes a ripple effect on societal welfare, which can be described as the welfare cost of default for households. In Dubey et al. (2000) and Dubey et al. (2005), default generates social benefit in the incomplete market setting by enlarging the asset span, while in Barro (1976), default causes net social welfare loss that is attributed to asymmetric information between the borrower and lender. In Barro's setting, loan default triggers a loss of collateral value for the borrower, making the lender's valuation far below the borrower's valuation during negotiations. The majority of research regarding corporate default cost has emphasized individual firms' profit loss (Warner, 1977; Altman, 1984; Weiss, 1990; Davydenko et al., 2012; Glover 2016). Households' welfare cost from default has not been thoroughly investigated, nor how monetary policy affects the welfare cost of default. Under CIA constraint, Cooley and Hansen (1989) estimated the welfare cost of inflation by comparing the steady states of two economies with differing money supply paths. This approach has the advantage of estimating welfare costs under various monetary policies. Building on this concept, this study measures households' welfare cost from default as the percentage change in real consumption required to maintain the same level of utility as the benchmark economy without default.

(Contributions)

To the best of our knowledge, this article is the first attempt to investigate the implications of corporate default and associated welfare costs from major monetary policies as an equilibrium phenomenon. Furthermore, this study contributes to the research on CIA models by examining the limitations of standard CIA models. Standard CIA models are not feasible for generating a positive response of output to an unexpected money growth shock (Schorfheide 2000). To overcome this limitation, we use a nonlinear form of nonpecuniary default penalty. The nonlinear default penalty causes the default rate to lower, which boosts lending in the corporate loan market, resulting in higher output. Finally, the findings of our study can assist policymakers in determining how to implement monetary policies to address a financial crisis characterized by high default rates.

2 The model

2.1 Cash-in-advance model structure

This model extends the CIA monetary dynamic stochastic general equilibrium (DSGE) model presented in Schorfheide (2000). This economy includes two types of agents, representative households and firms. Each household is identical to others, as is each firm. Both the representative household and firm are modeled as entities that exist indefinitely. Households provide labor, inherit the entire money stock from the previous period, and derive utility from leisure and consumption. Firms employ labor and capital to produce and sell final goods, deriving utility from profits. Since firms are owned by households, all profits are distributed to households as dividends at the end of each period. Households are subjected to CIA constraints that require them to have sufficient cash on hand prior to purchasing consumer goods; therefore, consumer goods in this economy are effective "cash goods". The CIA constraint structure allows the economy to fully reflect the current period's money supply growth. Since money is fiat, holding idle cash does not generate any utility for agents, and agents will lend all idle cash to those who need it. Figure 1 illustrates the study's model economy. Households and firms fulfill their roles in labor, corporate loan, and goods markets. The nominal price level (P_t) is determined by the interaction of demand and supply in the final goods market.

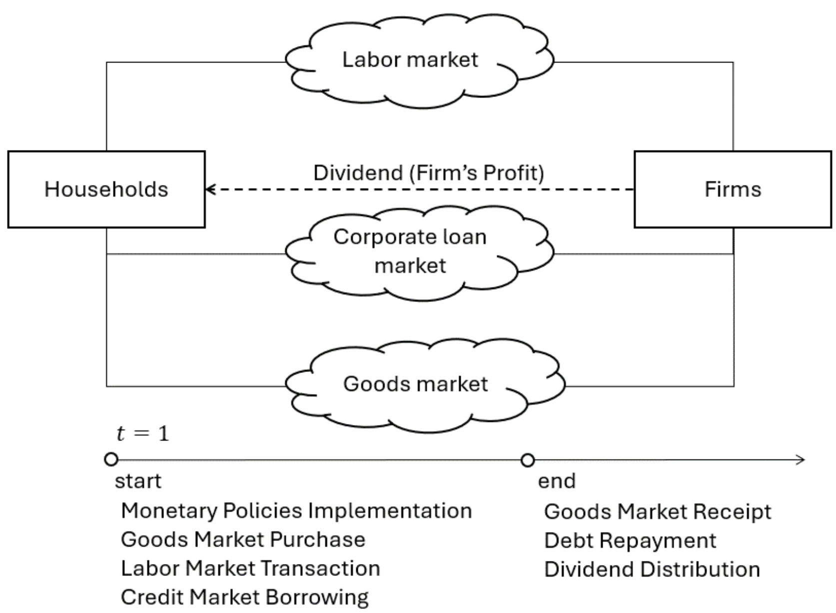

Figure 1: The model economy

At the beginning of period t, using inherited money, households determine the amount to allocate to purchasing final goods for current period consumption, the portion to lend to firms at the interest rate R_t, and the sum to be reserved for economic activities in the next period. Since firms distribute all profits to households at the end of each period, they begin each new period without no residual funds; therefore, firms must secure loans from households through the corporate loan market to hire workers. Labor contracts are signed with households, and wages are paid using funds obtained from the corporate loan market. Using labor and capital, firms produce and sell final goods in the goods market. In each period, firms must also decide the quantity of produced goods to allocate as an investment for the subsequent production period. To incorporate the concept of default, firms are permitted to default on their debts if they deem full repayment unworthy. They have the option to choose optimal repayment rates that maximize their utility. In the event of a default, the firm incurs a nonpecuniary penalty, which is reflected in its utility function and increases exponentially with the real size of the default.[2]

2.2 Households

Households engage in work, consumption, and money lending to firms. In each period, the representative household determines the optimal quantity of goods for consumption (C_t), labor supply (H_t), cash holding for the next period ($m_(t+1)$), and investment in the corporate loan market (the amount of money lent to firms) (D_t) to maximize the expected sum of discounted lifetime utility. The optimization problem is consequently solved as follows:

$$\max_{\{C_t, H_t, m_{t+1}, D_t\}} \quad \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \{(1-\phi)\ln C_t + \phi \ln(1-H_t)\},$$

s.t.

$$P_t C_t \leq m_t - D_t + W_t H_t + (g_t - 1) M_t, \tag{1}$$

$$0 \leq D_t,$$

$$m_{t+1} = (m_t - D_t + W_t H_t + (g_t - 1) M_t - P_t C_t) + \nu_t R_t D_t + F_t, \tag{2}$$

where $E_0(\cdot)$ is the expectation operator conditional on date 0, β represents the discount factor, ϕ is the preference shifter that reflects the marginal rate of substitution (MRS) between leisure and consumption, ν_t is the repayment rate determined by the firm, W_t is the nominal wage, F_t is the dividend from firms, M_t denotes the per capita money supply in period $t-1$, and the money stock in this economy follows a law of motion, $M_{t+1} = g_t M_t$.

The first constraint the household faces is the CIA constraint. The household has money inflow from the previous period's money stock inheritance (m_t), the current period's quantitative easing ($(g_t - 1)M_t$), and wage revenue ($W_t H_t$). At the same time, the household lends money (D_t) to the firm and purchases consumption goods with expenditure ($P_t C_t$). The money used for consumption cannot exceed the household's money balance ($m_t - D_t + W_t H_t + (g_t - 1)M_t$), indicating the CIA constraint. We provide a more detailed explanation of quantitative easing in subsection 2.5. The second constraint is that the representative household cannot borrow money from firms, and the household has no outstanding debts. The third constraint is the intertemporal budget, wherein the household receives money from the firm at the end of each period, in addition to the remaining money balance following consumption. As a creditor, the household receives repayment from the firm in the amount of $\nu_t R_t D_t$, and as an owner, the household receives a cash dividend distribution in the amount of F_t.

[2] Please see Appendix B for the model without default.

2.3 Firms

Firms initially obtain loans from households to employ workers and manufacture goods. At the start of each period, firms issue corporate bonds to fund the required workforce and repay an optimal fraction of outstanding debts at the end of the period. Notably, firms have the option to default on loans, subject to a nonpecuniary penalty. Firms determine the optimal repayment rate by assessing the benefits against the costs of default.

Firms use labor and capital to produce goods following a Cobb–Douglas production function:

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha},$$

where K_t denotes the capital stock which is predetermined at the beginning of period t, A_t is a labor-augmenting technology, and N_t represents labor input. α and $(1-\alpha)$ represent output elasticity of capital and labor, respectively.

Capital is accumulated according to the law of motion $K_{t+1} = (1-\delta)K_t + I_t$ $(0 < \delta < 1)$, where I_t is the new investment in current period. In each period, the representative firm makes optimal decisions on labor demand (N_t), bond issued (L_t), repayment rate (ν_t), nominal cash dividends (F_t), and the next period's capital stock (K_{t+1}). Firms distribute all profits to households at the end of each period; thus, the nominal dividends for the current period are discounted by the marginal utility of consumption in the next period. Additionally, firms are concerned about potential default penalties that could be levied.

In summary, the firm faces the following optimization problem:

$$\max_{\{F_t, K_{t+1}, N_t, L_t, \nu_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left\{ \frac{F_t}{C_{t+1}P_{t+1}} - \frac{\iota}{1+\eta} \left[(1-\nu_t) R_t \frac{L_t}{M_t} \right]^{1+\eta} \right\},$$

s.t.

$$W_t N_t \leq L_t, \tag{3}$$

$$F_t = L_t + P_t \left[K_t^{\alpha} (A_t N_t)^{1-\alpha} - K_{t+1} + (1-\delta) K_t \right] - W_t N_t - \nu_t R_t L_t, \tag{4}$$

where ι and η respectively denote the coefficient and the elasticity of nonpecuniary default for firms. When ι is higher, credit market regulation is stricter, and firms will subsequently hesitate to consider default. When η is higher, the default penalty rises more quickly with the real size of default, and the firm is more concerned regarding the negative effects of default. Therefore, ι can be described as the degree of default regulation and η can be described as the firm's degree of risk-aversion.

The first constraint indicates that the firm finances workers' wage payments at the beginning of each period by borrowing from households via corporate bonds. The second constraint shows that the firm earns sales revenue, repays $\nu_t R_t L_t$ to households, reinvests for the next production period, and distributes the remaining cash to households as dividends.

Technology is assumed to maintain the following stationary AR(1) process:

$$\ln A_t = \rho_A \ln A_{t-1} + (1-\rho_A) \ln \bar{A} + \sigma_A \varepsilon_{A,t},$$

$$\varepsilon_{A,t} \sim \text{i.i.d.}\, N(0,1),$$

where ρ_A refers to the AR(1) coefficient of technology, and $\ln \bar{A}$ indicates technology's steady state. Innovation ($\varepsilon_{A,t}$) follows a N$(0,1)$ process, where σ_A denotes the standard deviation of innovations to $\ln A_t$.

2.4 Default and penalties

In the standard general equilibrium framework, all agents honor promises and repay all outstanding debts. This study introduces agency problems of broken promises into a dynamic general equilibrium model. In an economy where default is allowed, borrowers make repayment decisions based on the tradeoff between the potential costs and benefits of default that means if borrowers' benefits outweigh the costs, a borrower might choose to default, even if this decision harms the lender. The default penalty is modeled as nonpecuniary form referencing Shubik and Wilson (1977), Tsomocos (2003), and Dubey et al. (2005).

The model assumes that the default penalty increases with the real amount of default (Tsomocos, 2003), and the nonpecuniary default penalty can be described as $MU_{\hat{l}t}(1-\nu_t)R_t\hat{L}_t$, where $(1-\nu_t)R_t\hat{L}_t$ represents the real value of default, and $MU_{\hat{l}t}$ is the marginal disutility of each real dollar of default on debts at time t.[3] Referencing Dubey et al. (2005), this study levies default penalties on borrowers who fail to keep their promises regardless of the reason. Therefore, when default levels are the same, defaulting firms that fail to honor debts due to ill fortune incur the same penalty as firms that default due to fraud.

Moreover, the marginal disutility of each real dollar default on debts varies with time, and we assume that $MU_{\hat{l}t}$ is a function of the real amount of default. In this model, $MU_{\hat{l}t} = \frac{\iota}{1+\eta}\left[(1-\nu_t)R_t\hat{L}_t\right]^{\eta}$ for firms, where ι is the constant coefficient of default penalty. Therefore, this study constructs the default penalty for borrowers as $\frac{\iota}{1+\eta}\left[(1-\nu_t)R_t\hat{L}_t\right]^{1+\eta}$. With this form of default penalty, the marginal disutility of each real dollar default rises with the real default amount; therefore, the penalty for default is stricter when the real default level is higher.

As noted previously, this study modifies the limitations of standard CIA models to generate plausible results when a positive monetary shock occurs. With the given coefficient of default penalty (ι), the elasticity of default penalty (η) can be adjusted to generate a positive output response when a positive monetary shock occurs (see Appendix E).

2.5 Monetary policy

Quantitative easing describes the Fed's large-scale asset purchases, which are intended to change the size and composition of the asset side of its balance sheet to support credit markets. In our CIA economy, the model does not differentiate the types of securities issued by defaultable private companies or default-free governments, instead focusing on explosive growth of money supply in the market. In this setup, economic outcomes can be determined under the quantity theory of money. The money growth rate in the current period is defined as $g_t = M_{t+1}/M_t$, where M_t is the money supply in period t. In each period, the central bank injects $(g_t-1)M_t$ money into the economy. To make our analysis simple and straightforward, we assume that the intertemporal money growth rate is stable in the long run and the log-linearized forms of g_t maintain the following stochastic AR(1) process:

$$\ln g_t = \rho_g \ln g_{t-1} + (1-\rho_g)\ln\bar{g} + \chi\sigma_{g,t}\varepsilon_{g,t}, \tag{5}$$

where $\bar{g}$ is unconditional mean, $\varepsilon_{g,t}$ is the innovation of quantitative easing, which follows a standard normal process, and $\sigma_{g,t}$ measures the standard deviation of monetary innovation. The model indicates that a more volatile $\sigma_{g,t}$ captures the confusion concerning the intentions of the central bank in terms of quantitative easing. Therefore, the last term ($\chi\sigma_{g,t}\varepsilon_{g,t}$) represents the quantitative easing resulting from the tone of forward guidance, which ranges from explicit to ambivalent.

Ambiguous forward guidance refers to qualitative phrases (less-explicit criteria) of monetary policy that do not indicate its specific duration or state-contingent terms such as "extended period," "moderately," and "substantial further progress" (Bernanke, 2022). Furthermore, the central bank sometimes weakens its message by expressing skepticism regarding the effectiveness of quantitative

[3] $\hat{L}_t$ is the detrended form. Please see Appendix A for details.

easing. These ambiguities introduce uncertainty concerning the size and duration of the quantitative easing, potentially resulting in increased $\sigma_{g,t}$. In contrast, explicit guidance with more precise phrases like "until maximum employment is achieved" or "until inflation has risen to 2%" can mitigate policy volatility and uncertainty (Bernanke, 2022). We assume that $\sigma_{g,t}$ maintains the following autoregressive processes (Fernández-Villaverde, 2010):

$$\ln\sigma_{g,t} = \rho_\sigma \ln\sigma_{g,t-1} + (1-\rho_\sigma)\ln\bar{\sigma}_g + \eta_\sigma \varepsilon_{\sigma,t}, \tag{6}$$

$$\varepsilon_{\sigma,t} \sim \text{i.i.d.}\, N(0,1),$$

where $\eta_\sigma \varepsilon_{\sigma,t}$ is the idiosyncratic shock concerning the ambiguity of forward guidance, ρ_σ is the coefficient of ambiguity of forward guidance, and $\bar{\sigma}_g$ is the standard deviation of monetary policy shock in the steady state. We describe the details of the estimations for equation (5) and (6) in Appendix C and D, respectively.

3 Equilibrium analysis

3.1 Market clearing conditions

This economy includes labor, corporate loan, and final goods markets. We assume that all markets are competitive. Thus, an equilibrium requires clearing all three markets in the model economies. The labor market clears when households' supply of labor is equal to firm's labor demand; therefore, $H_t = N_t$ holds for $\forall t \in T$.

For the corporate loan market, the amount of money that households are willing to lend equals the amount of money that firms need to borrow; therefore, $D_t = L_t$ holds for $\forall t \in T$. The market clearing condition for the final goods market is that the final goods demand for consumption and investment equals the supply of final goods produced by firms; therefore, $C_t + I_t = Y_t$ holds for $\forall t \in T$. This market clearing condition can be derived from households' (2) and firms' (4) budget constraints, and labor and money market equilibrium conditions.

Recall that all household and firm budget constraints are binding. In an equilibrium, when we combine budget constraints (1) and (3),

$$P_tC_t = m_t - L_t + W_tN_t + (g_t - 1)M_t,$$

$$W_tN_t = L_t.$$

In equilibrium, the money supply is optimized to equal money demand, thus $M_t = m_t$. This implies the following condition in the money market:

$$P_tC_t = M_{t+1}. \tag{7}$$

According to equation (7), money demand, which is denoted by money supply M_{t+1}, should be equal to consumption demand P_tC_t.

3.2 Optimality conditions

This section presents the results regarding the optimality conditions derived from the optimization problems faced by households and firms, providing insights into the characteristics of the CIA economy with default. The optimal condition for the money market (7) shows that money is neutral in this model economy in the long run, which aligns with the argument of real business cycle (RBC) theory and New Keynesian literature. However, money is not neutral in this artificial economy in the short run, in contrast to standard RBC theory.

It is imperative to clarify that the mechanism leading to short run money non-neutrality in this artificial CIA economy differs from that of a New Keynesian framework. In this economy, money non-neutrality is driven by transaction demands for money and the financing channel in which money and default have extensive influence, whereas real frictions such as monopolistic or asymmetric information are important under New Keynesian framework.

According to the FOCs derived from households' optimization problem, we can easily construct following equations:

$$\frac{\phi C_t P_t}{(1-\phi)(1-H_t)} - W_t = 0, \tag{8}$$

$$E_t\left(\frac{\beta \nu_t R_t}{C_{t+1}P_{t+1}}\right) - \frac{1}{C_t P_t} = 0. \tag{9}$$

Optimality condition (8) shows that the MRS between consumption and leisure must be equal to the nominal wage rate. Optimality condition (9) represents the stochastic consumption Euler equation, where the marginal utility of consumption in the current period should be equal to the expected weighted marginal utility of consumption in the next period. This means if households do not consume today and lend out the money, the marginal utility of consumption today is equivalent to the discounted marginal utility of next period's consumption that is enabled by the money received from repayment.

The optimality condition can be derived by combining the firm's borrowing constraint ($L_t = W_t N_t$), labor market equilibrium ($H_t = N_t$), where condition (8) becomes the following intertemporal labor market optimality condition:

$$\frac{\phi C_t P_t}{(1-\phi)(1-N_t)} - \frac{L_t}{N_t} = 0. \tag{10}$$

Equation (10) suggests that the labor market optimality condition is closely associated with the credit market (corporate loan market) and households' consumption decisions. One possible rationale is that firms must borrow money through the credit (corporate loan) market to pay wages; therefore, firms' financing ability can affect the demand side of the labor market.

For the firm's maximization problem, we construct the following optimality conditions by combining FOCs and equilibrium conditions:

$$\mathrm{E}_t\left[\frac{P_t}{C_{t+1}P_{t+1}} - \frac{\beta P_{t+1}}{C_{t+2}P_{t+2}}\left(\alpha K_{t+1}^{\alpha-1}\left(A_{t+1}N_{t+1}\right)^{1-\alpha} + (1-\delta)\right)\right] = 0, \tag{11}$$

$$\frac{(1-\alpha)P_t Y_t}{L_t} - R_t = 0, \tag{12}$$

$$\frac{\iota\left[(1-\nu_t)R_t L_t\right]^{\eta}}{M_t^{1+\eta}} - \mathrm{E}_t\left(\frac{1}{C_{t+1}P_{t+1}}\right) = 0. \tag{13}$$

The first condition (11) is the household's optimality condition for investment and consumption, indicating the tradeoff the economy faces regarding whether to invest more and delay consumption to the next period or consume today.

The second condition (12) is the equilibrium interest rate, which is determined by the firm's borrowing decision. Firms earn nominal sales revenue that is generated by labor inputs equal to the nominal cost of labor, which is denoted by financing cost for wages.

The third condition (13) identifies how firms make the optimal default decision. Pros and cons exist for defaulting. When firms default, they do not honor their obligations and subsequently have more cash on hand. Therefore, firms will distribute more cash dividends to households, which enables households to consume more. However, default firms incur a certain amount of nonpecuniary default penalty which increases nonlinearly with the real amount of default.

Proposition 1: Fisher Effect

For the artificial economy, suppose that the economy works in both goods and corporate loan markets, i.e., household's consumption ($C_t > 0$) and the repayment rate ($0 \leq \nu_t \leq 1$) for $\forall t \in T$. Then we can construct the following short-run equilibrium condition by taking the logarithm for Euler equations (9):

$$\ln R_t = \ln \mathrm{E}_t \left(\frac{U'_{C_t}}{\beta U'_{C_{t+1}}} \right) + \ln \mathrm{E}_t \left(\pi_{t+1} \right) + \ln \frac{1}{\nu_t}.$$

This indicates that the logarithmic form of the nominal interest rate is equal to the real interest rate plus inflation and the risk premium. Note that default risk is also considered in this model. The Fisher Effect proposition identifies the key factors that are closely correlated with the nominal variables, including consumption, inflation, and default risk premium. It also demonstrates that when nominal economic variables are affected, real variables will also be affected in terms of allocation.

Proposition 2: Quantity Theory of Money

In the model, the FOCs, binding CIA constraints, and budget constraints imply the following equilibrium condition:

$$P_t C_t = M_{t+1}.$$

The quantity theory of money proposition verifies long run money neutrality. In the long term, the economy converges to steady state in which $\frac{PY}{M}$ is constant.[4]

However, in dynamic terms, we obtain the following condition derived from the CIA constraints of two agents and goods market clearing conditions:

$$\frac{P_t Y_t}{M_{t+1}} = 1 + \frac{P_t I_t}{M_{t+1}}.$$

This indicates that the investment decision is influenced by monetary policy and this distortion is also transmitted into the real economy. Monetary factors have significant effects on real economic activities, confirming the non-trivial role of money. In the short run, the quantity theory of money proposition does not hold, and money is not neutral in this economy.

Corollary 1: Money Non-neutrality

Since money is fiat, agents hold money balances at the cost of foregone interest. To make the corporate loan market function, a positive interest rate is assigned that reduces the efficiency of trade and transactions; therefore, monetary policy is non-neutral.

[4]The quantity theory of money is expressed as $MV = PY$, where M refers to nominal money supply, P is price level, Y is real output, and V is the velocity of money circulation. It is usually assumed that V is relatively constant; thus, the quantity theory of money contends that the growth rate of price, plus the growth rate of output is equal to that of money supply.

Proposition 3: On-the-Verge Conditions

In any equilibrium of our benchmark economy, the detrended form of the following equation can be directly derived from the optimality condition (13) for $\forall t \in T$:

$$\mathrm{E}_t\left(\frac{1}{C_{t+1}\hat{P}_{t+1}g_t}\right) = \iota\left[(1-\nu_t)\,R_t\hat{L}_t\right]^{\eta}.$$

The on-the-verge condition implies that the firm will increase the repayment rate until the marginal utility of default equals the marginal disutility of default.

3.3 Households' welfare costs from corporate default

The welfare costs of corporate default for households are determined by comparing the steady states of models with various default levels. Cooley and Hansen (1989) took a similar approach to measure the cost of the inflation tax. In detail, this study's measure of welfare cost is based on increased consumption that the representative household would require in the economy with default to be as well off as in the basic CIA economy without default. First, we get ΔC from the following equation for further welfare calculation:

$$\bar{U} = \left[(1-\phi)\ln\left(C^{*}+\Delta C\right) + \phi\ln\left(1-H_t^{*}\right)\right],$$

where $\bar{U}$ is the maximized utility level in the steady state of the economy without default, while C^{*} and H^{*} are the respective optimal consumption and labor (working time) in the steady state of the economy with default.

In this study, ΔC represents the consumption compensation (loss) for the representative household in the economy with default. If ΔC is positive, households in the economy with default should be compensated with positive consumption to be as well off as in the economy without default; otherwise, they are worse off, and the default option causes welfare loss. If ΔC is negative, households in the economy with default can be as well off as in the economy without default even if they consume less than the current optimal consumption level. In this case, the default option brings welfare improvement.

Furthermore, to determine the size of the economy, this study references Cooley and Hansen (1989), taking the ratio of consumption compensation to steady state real output ($\Delta C/Y$) and the ratio of consumption compensation to steady state real consumption ($\Delta C/C$) as the measure of the welfare cost.

4 Calibration

4.1 Parameters

We solve the model numerically, given its complexity, fixing several parameter values at a quarterly frequency. Our criterion for selecting parameters was to use conventional values from previous research. We choose the discount factor $\beta = 0.99$ as a default choice. For the depreciation rate, $\delta = 0.025$ is set to induce the appropriate capital–output ratio (Fernández-Villaverde et al., 2010). We use $\alpha = 0.32$ for the capital share in the US production function (Schmitt-Grohé and Uribe, 2003) and set the AR(1) coefficient of technology to $\rho_A = 0.95$ with a standard deviation of innovation of $\sigma_A = 0.007$, following Cooley and Prescott (1995). We further set the value of total factor productivity in the steady state to $\bar{A} = 1$. The AR(1) coefficient of the growth rate of monetary injection, $\rho_g = 0.6488$, and the smoothing coefficient of the ambiguity in forward guidance, $\rho_\sigma = 0.9435$, are directly estimated from the data (see Appendix D). Nason and Cogley (1994) estimated a MRS

Table 1: Implied parameters

Description	Parameter	Value
Output elasticity of capital	α	0.3200
Discount factor	β	0.9900
Consumption and leisure preference shifter	ϕ	0.7730
Depreciation rate	δ	0.0250
Coefficient of default penalty	ι	150
Elasticity of nonpecuniary default	η	3
Steady state labor-augmenting technology factor	$\bar{A}$	1
AR(1) coefficient of technology	ρ_A	0.9500
AR(1) coefficient of money supply growth	ρ_g	0.6488
Smoothing coefficients of ambiguity in forward guidance	ρ_σ	0.9435
Standard deviation of technology innovation	σ_A	0.0070

of $\phi = 0.773$, and we use this value for our model calibration. Therefore, all the parameters are well within the range of values used in previous macroeconomic literature. We apply the reverse engineering method by calibrating the steady state for parameters such as ι and η (see Appendix E). Table 1 presents all of these implied parameters.

4.2 Steady state and stability

To further examine the stability of our steady state for the endogenous variables, we compare the absolute values of the simultaneous equations with the solutions specifying the minimum values of all the variables by orders of magnitude to determine the calculation error of the steady state. We then draw the Figure 2 for non-default and default models to incorporate the models without and with default, respectively. As all errors of the simultaneous equation in steady state are considerably far in magnitude from the computation error, this confirms that our steady state is quite stable.

5 Results

5.1 Steady state implications of default

The steady state of all endogenous variables is summarized in Table 2. The repayment rate (ν) sets the range of $[0,1]$.[5] Note that the gross interest rate in steady state ($\bar{R}$) in the default model is larger than that in the non-default model. This can be explained by the Fisher Effect (Proposition 1), where in the steady state, as the price does not change over time, inflation risk is not present; therefore, the only difference is the default risk between non-default and default models, which is denoted by $\ln \bar{\nu}^{-1}$.

Furthermore, the default model presents differences in the steady state compared with the non-default model. Therefore, we are able to determine the role of default by analyzing such variations. First, when agents in the benchmark economy are allowed to default, firms will choose optimal repayment rate (ν) to maximize the expected utility profit ($\bar{F}$); therefore, firms' repayment rate deviates from one and the firm increases its expected profit. Meanwhile, the gross interest rate (R) increases as the risk premium begins to emerge due to default risk. Under these circumstances, firms

[5] Firms do not repay more than what they own ($\nu \leq 1$) and are not rewarded for defaulting on obligations ($\nu \geq 0$).

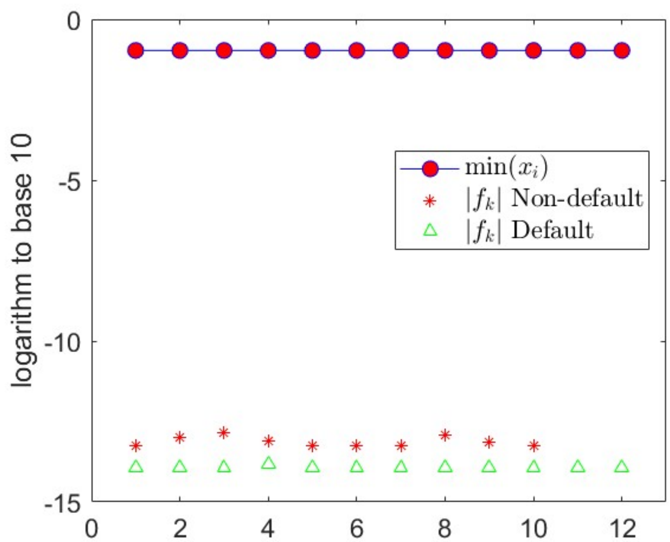

Figure 2: Calculation errors of benchmark economy in the steady state

Note: x_i is an endogenous variable for $i = 1, 2, \ldots, n$. $f_k = 0$ is the k^{th} simultaneous equation in the steady state for $k = 1, 2, \ldots, n$. Since $\forall |f_k| \leq 10^{-12}$ and $\min(x_i) \sim 10^{-1}$, this confirms that the steady state in our calibration exercise is stable.

Table 2: Steady state of the model

Description	Parameter	Non-default	Default
Monetary growth rate	$\bar{g}$	1.0151	1.0151
Output	$\bar{Y}$	0.5699	0.4709
Price of goods	$\bar{P}$	2.3069	2.7920
Wage per worker	$\bar{W}$	4.3286	4.1469
Labor used for production	$\bar{N}$	0.2014	0.1664
Consumption	$\bar{C}$	0.4400	0.3636
Capital used for production	$\bar{K}$	5.1957	4.2930
Loans to firms	$\bar{L}$	0.8719	0.6902
Interest rate	$\bar{R}$	1.0254	1.2953
Firm profit	$\bar{F}$	0.1211	0.3074
Investment	$\bar{I}$	0.1299	0.1073
Repayment rate	$\bar{\nu}$	1	0.7916

cannot borrow an adequate amount due to high financing cost. As a result, firms borrow less in loans ($\bar{L}$). Second, the default begins to have an impact on the real economy, liquidity dries up, firms lose opportunities to invest, and production activity could be weakened, and as output ($\bar{Y}$) decreases, so too the labor ($\bar{N}$) and capital ($\bar{K}$) used for production. Third, price ($\bar{P}$) rises since the supply of goods decreases, wages per worker decline because of lowered labor demand, and households consume less because of lower income and higher prices, resulting in a decreased consumption ($\bar{C}$). This implies that household welfare could decline as a result of firm default. In conclusion, the gap between the steady state in the default model and that in the non-default model aligns with stylized facts, indicating the validity of our default model calibration exercise.

5.2 Impact of quantitative easing

Cyclical property of quantitative easing

To investigate the effects of expansionary monetary policy, we illustrate in the impulse response functions (IRFs) to a positive money growth rate shock as shown in Figure 3, which reveals that money growth shock has real effects on output and consumption as money is necessary to consume (i.e., CIA constraint). Thus, the non-trivial role of money is confirmed. Furthermore, money growth shock decreases the default risk. First, quantitative easing causes inflation and higher prices. In response, households work harder and consume less to offset the adverse effect of inflation. The same logic applies when households choose to deposit more for precautionary purposes, resulting in more loan supplies. Accordingly, firms invest more in capital through financing from credit markets and hire more labor to optimally produce by matching the marginal utility of capital to that of labor. As a result, money growth leads to more output in the CIA model with default.

As the nominal interest rate is approximately equal to the real interest rate plus expected inflation (Fisher Effect, Proposition 1), it increases slightly. Comparing the two models, the default model exhibits minimal growth in the nominal interest rate, in contrast to the volatile fluctuations observed in the non-default model. Empirically, the Fed's approach to quantitative easing, involving the substantial purchase of long-term government bonds and government-guaranteed, mortgage-backed securities, is intended to effectively reduce long-term interest rates (Bernanke, 2022). The gradual growth of the nominal interest rate in the model with endogenous default is a plausible response to quantitative easing.

The change in repayment rate can be explained by on-the-verge conditions (Proposition 3). The marginal utility of default decreases due to higher prices relative to consumption, while the marginal disutility of default tends to rise with more corporate loan supplies and a higher nominal interest rate. The repayment rate must increase for on-the-verge conditions (Proposition 3) to hold. During the 2008 financial crisis, part of the solution was to devalue the currency by increasing the money supply, which made at least some debts easier to repay (Jones, 2014).

Comparing the two models with and without default demonstrates the role of corporate default and its interaction with liquidity, revealing the transfer mechanism of quantitative easing in the real economy. Recall that in standard CIA models, output negatively responds to the positive monetary shock (Schorfheide, 2000). However, in this default model, it successfully generates a plausible response of output from quantitative easing, at least in the short-run. During economic recession, as witnessed in the 2008 global financial crisis and the COVID-19 pandemic, quantitative easing policies provide a wide range of benefits to advance economic recovery such as increasing employment, wages, capital investment, and tax revenue (Bernanke, 2022). Our default model supports the positive effect of this approach on macroeconomic factors relative to the non-default case.

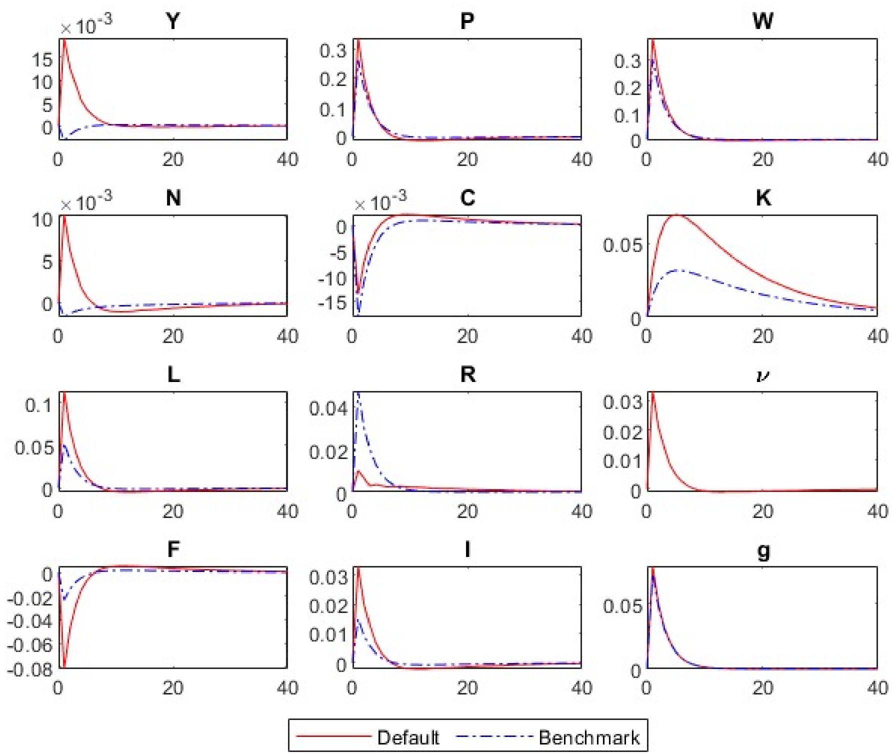

Figure 3: IRFs of quantitative easing

Welfare cost of default with quantitative easing

The economy in our model simulates three scenarios associated with varying quantitative easing, including a low money growth rate ($g = 1.0015$), a medium money growth rate ($g = 1.0151$), and a high money growth rate ($g = 1.0286$). The three different money growth rates are chosen based on the basic statistics of actual US M1 data (see the note on Table 3). In each scenario, three sub-scenarios stretching varying default levels are explained in detail, including (1) low default (high repayment; $\nu = 0.9972$), (2) medium default (medium repayment; $\nu = 0.9945$), and (3) high default (low repayment; $\nu = 0.9910$).

The welfare costs of default are presented in the bottom rows of each panel in Table 3. For the medium money growth rate with medium default ($g = 1.0015$; $\nu = 0.9945$), the welfare cost of default is estimated as 0.0488% of real output ($\Delta C/Y$), and welfare loss has considerable economic significance. In 2019 Q4, the US real Gross National Product (GNP), measured in chained 2012 dollars, was $19,445 billion.[6] At this level, the cost would amount to more than $9 billion.

For the low money growth rate with low default ($g = 1.0015$; $\nu = 0.9972$), the welfare cost of default is 0.0225% of real output. At the same money growth rate, the welfare cost corresponding to medium default ($\nu = 0.9945$) increases to 0.0447% of real output, and for high default ($\nu = 0.9910$), welfare cost increases to 0.0742% of real output, as shown in the top panel of Table 3. In panels 2 and 3, welfare costs consistently increase with the severity of default.

First, the low default penalty decreases the marginal disutility of default; therefore, firms will default more when facing loose regulations to increase profit. Accordingly, firms will produce more and distribute more dividends to households, driving up consumption in optimal allocation. However, according to Fisher Effect (Proposition 1), a higher default rate requires a higher risk premium, which pushes up an interest rate in the corporate loan market. With higher financing costs, firms demand fewer corporate loans, which reduces their investment and production and lowers consumption due to the income effect. Higher marginal utility of consumption subsequently increases the default rate through on-the-verge conditions (Proposition 3), which amplifies the effect of default penalties. Consequently, the second effect dominates the first one.

Furthermore, at the same default level, the cost of default is higher in the economy with intensive quantitative easing as demonstrated by the results of panels 1–3 of Table 3. At default level $\nu = 0.9972$, the welfare cost of default is 0.0225% of real output when the expected money growth rate is low ($g = 1.0015$). When growth rate of the money supply is higher ($g = 1.0151$), welfare cost correspondingly increases to 0.0246% of real output. Therefore, firms' default causes higher welfare loss at the same level of repayment rate when the money growth rate is higher.

At a higher money growth rate, the nominal interest rate is higher at the same default rate. Considering the Fisher Effect (Proposition 1), the change in inflation is greater than the change in real interest rate. More money supply pushes up prices, with a relatively constant corporate loan amount, and consumption decreases, indicating on-the-verge conditions (Proposition 3). Accordingly, the nominal interest rate in this economy increases with a higher marginal utility of consumption, which further diminishes the consumption stream.

In the cyclical properties of our benchmark economy demonstrate that at the same level of credit regulation, when money supply increases, the default rate will decrease. This supports the rationale behind the central bank's decision to inject money into the economy during the financial crisis. However, as shown above, this expansionary monetary policy has adverse effects, resulting in a greater welfare cost of default compared with the same level of default.

[6] US real GNP data are sourced from the US Bureau of Economic Analysis.

Table 3: Welfare cost of default with varying degrees of quantitative easing

	ν	1.0000	0.9972	0.9945	0.9910
$g = 1.0015$	Output	0.5761	0.5748	0.5736	0.5719
	Price	2.2516	2.2567	2.2616	2.2679
	Wage	4.2823	4.2799	4.2775	4.2745
	Labor	0.2036	0.2032	0.2027	0.2021
	Consumption	0.4448	0.4438	0.4428	0.4416
	Capital Stock	5.2519	5.2402	5.2288	5.2142
	Corporate Loan	0.8719	0.8695	0.8671	0.8641
	Nominal Interest Rate	1.0116	1.0145	1.0172	1.0208
	Real Interest Rate	1.0101	1.0129	1.0157	1.0193
	Firm Profit	0.1194	0.1219	0.1243	0.1274
Welfare Cost	$\Delta C/C \times 100$	0.0000	0.0292	0.0579	0.0961
	$\Delta C/Y \times 100$	0.0000	0.0225	0.0447	0.0742
$g = 1.0151$	Output	0.5699	0.5686	0.5674	0.5658
	Price	2.3069	2.3121	2.3171	2.3236
	Wage	4.3286	4.3262	4.3238	4.3208
	Labor	0.2014	0.2010	0.2005	0.2000
	Consumption	0.4400	0.4390	0.4381	0.4369
	Capital Stock	5.1957	5.1841	5.1728	5.1583
	Corporate Loan	0.8719	0.8695	0.8671	0.8641
	Nominal Interest Rate	1.0254	1.0282	1.0310	1.0347
	Real Interest Rate	1.0101	1.0129	1.0157	1.0193
	Firm Profit	0.1211	0.1236	0.1260	0.1291
Welfare Cost	$\Delta C/C \times 100$	0.0000	0.0319	0.0632	0.1048
	$\Delta C/Y \times 100$	0.0000	0.0246	0.0488	0.0809
$g = 1.0286$	Output	0.5639	0.5627	0.5614	0.5599
	Price	2.3624	2.3677	2.3729	2.3796
	Wage	4.3746	4.3722	4.3698	4.3668
	Labor	0.1993	0.1989	0.1984	0.1979
	Consumption	0.4354	0.4344	0.4335	0.4323
	Capital Stock	5.1411	5.1296	5.1184	5.1040
	Corporate Loan	0.8719	0.8695	0.8671	0.8641
	Nominal Interest Rate	1.0390	1.0419	1.0447	1.0484
	Real Interest Rate	1.0101	1.0129	1.0157	1.0193
	Firm Profit	0.1227	0.1252	0.1277	0.1308
Welfare Cost	$\Delta C/C \times 100$	0.0000	0.0345	0.0684	0.1132
	$\Delta C/Y \times 100$	0.0000	0.0266	0.0528	0.0874

Note: 1. This paper measures the corporate bond yield spread as the difference between corporate and 10-year treasury bonds. According to the Fisher Effect (Proposition 1), quarterly repayment rate ν approximately equals $e^{-\text{YieldSpread}/4}$. For the case of low default, we use Moody's Seasoned Aaa Corporate Bond Yield relative to the yield on 10-year treasury constant maturity data from 1970Q1 to 2019Q4. For the case of medium default, we use Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-year treasury constant maturity data from 1970Q1 to 2019Q4. For the case of high default, we use $\mu_{\text{Baa-10Y}} + 2\sigma_{\text{Baa-10Y}}$.

2. To imitate the conditions of low, medium, and high money growth rates, we use quarterly historical US M1 growth rate data from 1970Q1 to 2019Q4, with mean $\mu_g = 1.0151$ and standard deviation $\sigma_g = 0.0135$. The growth rate of money supply for each case is $(\mu_g - \sigma_g), (\mu_g)$, and $(\mu_g + \sigma_g)$, respectively.

3. Interest rate adjusted for inflation, i.e., real interest rate = nominal interest rate/g.

5.3 Impact of forward guidance

5.3.1 Cyclical property of forward guidance

We study the impact of explicit forward guidance (negative shock on σ_g) via IRFs, presenting the results in Figure 4. The analysis is divided into two stages to determine a transmission channel through which default could be influenced. In the first stage, we confirm the transmission channel between explicit forward guidance and recession recovery. The findings reveal that explicit forward guidance is procyclical, and lower uncertainty is accompanied by stimulating economy. The results are similar to previous research showing that policy uncertainty generally has a negative effect on the economy, as argued by Baker et al. (2012), Stokey (2013), Fernández-Villaverde et al. (2011), and Bloom (2013).

First, agents become less risk averse when faced with low monetary uncertainty. This change influences firms' and households' decisions. Low monetary policy uncertainty makes firms less concerned about immediate investment and labor inputs in the short run, as they are more certain of the future outcomes of their investment decisions; conversely, with uncertainty, firms may delay capital investment or hiring until they have more information about economic conditions (Bernanke, 2022). Capital stock increases as a result of increased investment. Consequently, households benefit from increased employment due to firms' higher investment, and choose to consume more and deposit less. This results in less loans because of decreased deposits, which decreases the wage rate. After that, we can expect a changing output pattern. Increased capital and labor drive output increases, and rising demand for goods also contributes to higher output. Furthermore, increased output results in higher consumption and investment. Therefore, explicit forward guidance leads to more output and stimulates the economy in the first stage.

In the second stage, society has entered a recovery phase, influencing firms' default and interest rates. According to the on-the-verge conditions (Proposition 3), the marginal utility of default decreases due to increased consumption. However, both R_t and $\hat{L}_t$ decrease, driving the marginal disutility of default decrease, which is higher than the decrease in the marginal utility of default. Thus, firms' default rate $(1-\nu_t)$ needs to increase to match the marginal utility and disutility of default; therefore, firms' default rate lowers in the second stage. A lower risk premium can be observed through the decreased default rate of firms. In the corporate loan market, decreased risk premium $(1/\nu_t)$ dominates the nominal interest rate R_t (Fisher Effect, Proposition 1), resulting in a fall in nominal interest rates, which further enhances investments.

Overall, explicit forward guidance can reduce the default rate and stimulate the economy without increasing the interest rate, in contrast to the effects of quantitative easing; however, in terms of magnitude, explicit forward guidance has a smaller impact on the economy compared with quantitative easing. A comparison between the models with and without default further emphasizes the role of corporate default in capturing the transmission mechanism of explicit forward guidance in the real economy. In contrast to the scenario without default, the default model has a positive impact on the economy, leading to increased output through firms' intensified investment, ultimately resulting in a decreased default rate.

5.3.2 Welfare cost of default with forward guidance

The economy in this study's model simulates three scenarios associated with different tones of forward guidance, including a strong degree of ambiguity ($\sigma_g = 0.0131$), a medium degree of ambiguity ($\sigma_g = 0.0101$), and a weak degree of ambiguity ($\sigma_g = 0.0071$), which are chose based on the US M1 data (see the note in Table 4). In each scenario, this study introduces three sub-scenarios of default variations, including (1) low default ($\nu = 0.9972$), (2) medium default ($\nu = 0.9945$), and

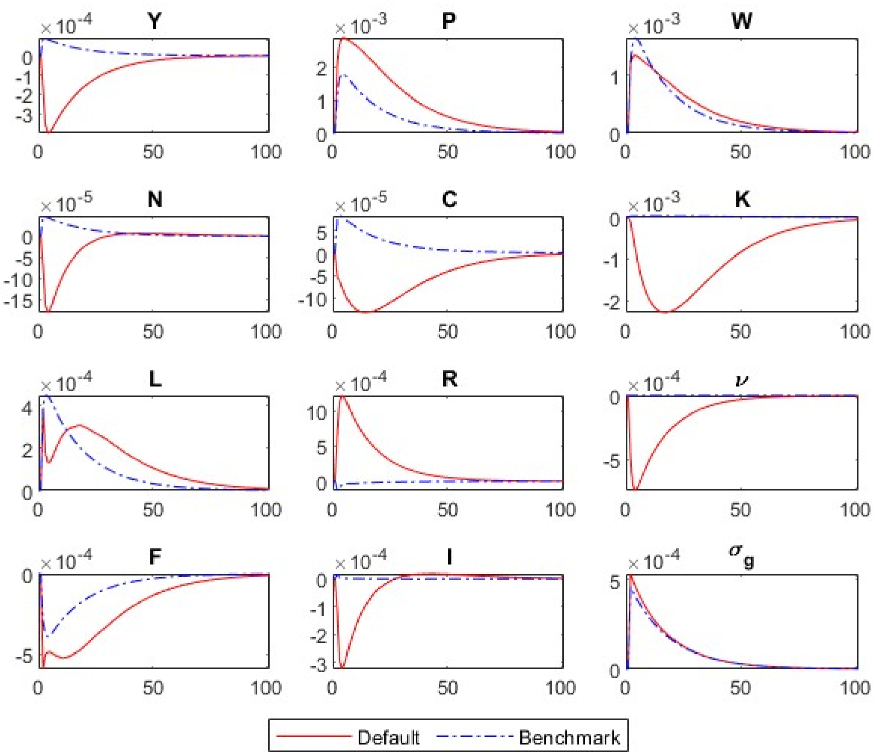

Figure 4: IRFs of explicit forward guidance

(3) high default ($\nu = 0.9910$).

The results of welfare analysis are presented in the bottom rows of each panel in Table 4. For the strong degree of ambiguity with low default ($\sigma_g = 0.0131, \nu = 0.9972$), the welfare cost of default is 0.0250% of real output. At the same uncertainty level, the welfare cost of medium default ($\nu = 0.9945$) increases to 0.0496% of real output, and that of high default ($\nu = 0.9910$) is 0.0822% of real output as shown in the top panel of Table 4.

At the same default level, the cost of default is lower in the economy with a weak degree of ambiguity by comparing the results in panels 1–3 of Table 4. At the low default level ν=0.9972, the welfare cost of default is 0.0250% of real output when expected uncertainty is high ($\sigma_g = 0.0131$). When uncertainty is lower ($\sigma_g = 0.0101$), welfare cost correspondingly decreases to 0.0249% of real output. Therefore, the presence of default causes welfare gains at the same level of repayment rate when monetary uncertainty is lower. Meanwhile, the nominal interest rate is lowered to the same level as the default rate. This can be further explained by Fisher Effect (Proposition 1), which offers an explanation opposing that of the quantitative easing policy in Table 3.

Cyclical economic properties show that when the central bank employs explicit forward guidance, the default rate will decrease at the same level of credit regulation. However, the contradiction with quantitative easing is that more explicit forward guidance steers the economy toward a financially less fragile regime, which subsequently results in a lower welfare cost of default for the same level of default.

Table 4: Welfare cost of default with various tones of forward guidance

		ν 1.0000	0.9972	0.9945	0.9910
$\sigma_g = 0.0131$	Output	0.5687	0.5674	0.5662	0.5646
	Price	2.3181	2.3233	2.3283	2.3349
	Wage	4.3379	4.3355	4.3331	4.3301
	Labor	0.2010	0.2006	0.2001	0.1996
	Consumption	0.4391	0.4381	0.4372	0.4359
	Capital Stock	5.1846	5.1730	5.1618	5.1472
	Corporate Loan	0.8719	0.8695	0.8671	0.8641
	Nominal Interest Rate	1.0281	1.0310	1.0338	1.0374
	Real Interest Rate	1.0101	1.0129	1.0157	1.0193
	Firm Profit	0.1214	0.1239	0.1263	0.1295
Welfare Cost	$\Delta C/C \times 100$	0.0000	0.0324	0.0643	0.1065
	$\Delta C/Y \times 100$	0.0000	0.0250	0.0496	0.0822
$\sigma_g = 0.0101$	Output	0.5690	0.5677	0.5665	0.5649
	Price	2.3155	2.3207	2.3257	2.3323
	Wage	4.3358	4.3333	4.3310	4.3279
	Labor	0.2011	0.2006	0.2002	0.1997
	Consumption	0.4393	0.4383	0.4374	0.4361
	Capital Stock	5.1871	5.1755	5.1643	5.1497
	Corporate Loan	0.8719	0.8695	0.8671	0.8641
	Nominal Interest Rate	1.0275	1.0304	1.0332	1.0368
	Real Interest Rate	1.0101	1.0129	1.0157	1.0193
	Firm Profit	0.1213	0.1238	0.1262	0.1294
Welfare Cost	$\Delta C/C \times 100$	0.0000	0.0323	0.0640	0.1061
	$\Delta C/Y \times 100$	0.0000	0.0249	0.0494	0.0819
$\sigma_g = 0.0071$	Output	0.5693	0.5680	0.5668	0.5652
	Price	2.3130	2.3181	2.3232	2.3297
	Wage	4.3337	4.3312	4.3289	4.3258
	Labor	0.2012	0.2007	0.2003	0.1997
	Consumption	0.4395	0.4385	0.4376	0.4363
	Capital Stock	5.1897	5.1780	5.1668	5.1523
	Corporate Loan	0.8719	0.8695	0.8671	0.8641
	Nominal Interest Rate	1.0268	1.0297	1.0325	1.0362
	Real Interest Rate	1.0101	1.0129	1.0157	1.0193
	Firm Profit	0.1212	0.1238	0.1262	0.1293
Welfare Cost	$\Delta C/C \times 100$	0.0000	0.0322	0.0638	0.1057
	$\Delta C/Y \times 100$	0.0000	0.0248	0.0492	0.0816

Note: 1. To imitate strong, medium, and weak degrees of forward guidance ambiguity, we generate the value of stochastic volatility of money growth rate at each period using quarterly historical US M1 growth rate data from 1970Q1 to 2019Q4 (Appendix D). The mean of σ_g is $\mu_{\sigma_g} = 0.0101$, and the standard deviation is $s_{\sigma_g} = 0.0015$. The monetary supply for each case is $(\mu_{\sigma_g} + 2s_{\sigma_g})$, (μ_{σ_g}), and $(\mu_{\sigma_g} - 2s_{\sigma_g})$.
2. Interest rate is adjusted for inflation, i.e., real interest rate = nominal interest rate$/g$.

6 Concluding remarks

This study employs a DSGE model that incorporate CIA constraints and endogenous corporate default to estimate how defaults and the welfare costs of default for households vary with monetary policies, including quantitative easing and explicit forward guidance. The findings demonstrate that both monetary policies stimulate the economy and decrease default risk; however, quantitative easing exacerbates households' welfare costs due to defaults, while explicit forward guidance mitigates them. The study reveals that the welfare cost of corporate default due to quantitative easing and forward guidance amounts to 0.0488 and 0.0494% of real output, respectively. Furthermore, we explore the cyclical properties of monetary policy, showing that quantitative easing leads to higher prices and interest rates, which influence firms' and households' decisions. In contrast, explicit forward guidance, characterized by low monetary uncertainty, boosts the economy by encouraging investment and lowering default and interest rates; however, its impact is limited in magnitude.

Endogenous default significantly not only influences short-term economic fluctuations, but also long-term real economic activities. When the CIA model incorporates nonpecuniary and nonlinear default penalties, sensitivity to fluctuations in monetary policy is amplified through credit market dynamics. This approach overcomes the limitations of standard CIA models by generating a positive relationship between output response and monetary shock.

The study's insights are crucial for policymakers in shaping monetary policy. The contrasting effects of quantitative easing and forward guidance on welfare costs highlight the need for a balanced approach in policy implementation. Central banks must consider these dynamics to optimize policy effectiveness, particularly in periods of financial instability. This study contributes to a deeper understanding of the interplay between monetary policies and corporate financial health to guide future policy directions.

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Appendices

Appendix A. List of detrended equations

Since money has stochastic trends and grows at a constant rate in the long run, the artificial economy in this study is non-stationary. First, all real variables ($q_t = [Y_t, C_t, I_t, K_{t+1}]$) and labor ($N_t$) are stationary because no technology or population growth. However, all nominal variables ($Q_t = [L_t, W_t, P_t]$) grow with the money supply (M_t). Therefore, all nominal variables in the economy should be detrended to obtain a steady state, dividing them by the money supply in this economy, i.e., $\hat{Q}_t = Q_t/M_t$ (Griffoli, 2007). Variables with hats represent stationary variables after detrending. Finally, the set of detrended equations in this model economy is as follows:

$$
\begin{aligned}
\ln A_t &= \rho_A \ln A_{t-1} + (1-\rho_A)\ln \bar{A} + \sigma_A \varepsilon_{A,t}, \\
\ln g_t &= \rho_g \ln g_{t-1} + (1-\rho_g)\ln \bar{g} + \chi \sigma_{g,t} \varepsilon_{g,t}, \\
\ln \sigma_{g,t} &= \rho_\sigma \ln \sigma_{g,t-1} + (1-\rho_\sigma)\ln \bar{\sigma}_g + \eta_\sigma \varepsilon_{\sigma,t}, \\
\mathrm{E}_t\left(\frac{\hat{P}_t}{C_{t+1}\hat{P}_{t+1}g_t}\right) &= \mathrm{E}_t\left(\frac{\beta \hat{P}_{t+1}}{C_{t+2}\hat{P}_{t+2}g_{t+1}}\left(\alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta)\right)\right), \\
\hat{W}_t &= \frac{\hat{L}_t}{N_t}, \\
\frac{\hat{L}_t}{N_t} &= \frac{\phi C_t \hat{P}_t}{(1-\phi)(1-N_t)}, \\
\frac{(1-\alpha)\hat{P}_t Y_t}{\hat{L}_t} &= R_t, \\
\frac{1}{R_t} &= \beta \mathrm{E}_t\left(\frac{\nu_t C_t \hat{P}_t}{C_{t+1}\hat{P}_{t+1}g_t}\right), \\
C_t + K_{t+1} &= (1-\delta)K_t + K_t^{\alpha}(A_t N_t)^{1-\alpha}, \\
\hat{P}_t C_t &= g_t, \\
Y_t &= K_t^{\alpha}(A_t N_t)^{1-\alpha}, \\
\mathrm{E}_t\left(\frac{1}{C_{t+1}\hat{P}_{t+1}g_t}\right) &= \iota\left[(1-\nu_t)R_t\hat{L}_t\right]^{\eta}.
\end{aligned}
$$

Appendix B. Basic CIA Economy without default

This appendix describes a basic CIA economy without default, which is the starting point of the analysis in this study. Default is not allowed in this economy, firms will pay back all debts to households, and there is no agency problem in this economy.

Households without default

Households solve the optimization problem as follows:

$$\max_{\{C_t, H_t, m_{t+1}, D_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \{(1-\phi)\ln C_t + \phi \ln(1-H_t)\},$$

s.t.

$$P_t C_t \leq m_t - D_t + W_t H_t + (g_t - 1) M_t, \tag{14}$$

$$0 \leq D_t, \tag{15}$$

$$m_{t+1} = (m_t - D_t + W_t H_t + (g_t - 1) M_t - P_t C_t) + R_t D_t + F_t. \tag{16}$$

Firms without default

Firms in this economy face the optimization problem as follows:

$$\max_{\{F_t, K_{t+1}, N_t, L_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left\{ \frac{F_t}{C_{t+1} P_{t+1}} \right\},$$

s.t.

$$W_t N_t \leq L_t, \tag{17}$$

$$F_t = L_t + P_t \left[K_t^{\alpha} (A_t N_t)^{1-\alpha} - K_{t+1} + (1-\delta) K_t \right] - W_t N_t - R_t L_t. \tag{18}$$

Appendix C. M1 money growth rate

Since the Organization for Economic Co-operation and Development provides quarterly money supply M1 data, we use the quarterly growth rate in one-time period forward divided by that in the current time period. Table 5 presents the estimate results for equation (5). The AR (1) coefficient and constant are statistically significant.

Table 5: Estimation of the AR(1) model for the growth rate of money supply

Description	Parameter	Standard error	p-value
Constant	0.0053	0.0011	$< 10^{-4}$
AR(1) coefficient ρ_g	0.6488	0.0543	$< 10^{-4}$
RMSE	0.0102	-	
R-square	0.42	-	

Based on the estimate result, we set smoothing coefficients of quantitative easing, $\rho_g = 0.6488$. The standard deviation of the quantitative easing shock is 0.0102. To obtain the value of χ, the variance of the residual is written as $\chi^2 \operatorname{var}(\sigma_{g,t}) = 0.0102^2$ as $\sigma_{g,t}$, and $\varepsilon_{g,t}$ is assumed to be independent. Notice that $\operatorname{var}(\sigma_{g,t}) = \exp\left(2\ln\bar{\sigma}_g + \frac{2\eta_\sigma^2}{1-\rho_\sigma^2}\right) - \exp\left(2\ln\bar{\sigma}_g + \frac{\eta_\sigma^2}{1-\rho_\sigma^2}\right)$.[7] Therefore, we can use this equation to determine that $\chi = 7.2018$ by plugging in the value of parameters. We can then easily calculate the value of constant money supply growth rate in steady state $\bar{g} = 1.0151$ by solving the equation, $(1-\rho_g)\ln\bar{g} = 0.0053$.

According to the Figure 4, the data and the model estimates move together and nearly simultaneously. In Figure 5, the outliers that occurred in the early 1980s and around 2008–2010 are reasonable due to extreme situations, whereas the majority of the data can be well-explained by the model. The US entered recession in January 1980 and returned to growth six months later in July 1980. Due to an unchanged unemployment rate, a second recession started in July 1981 (Bednarzik et al., 1982). The remaining impact of the energy crisis and contractionary monetary policy adopted by the Fed to combat double digit inflation were driving forces of the 1980 recession (Iden et al., 1982; Feldstein, 1994). The downturn ended 16 months later, in November 1982. The economy entered a strong recovery, with a lengthy expansion through 1990 (Gardner, 1994). The early 2000s was a decline in economic activity that primarily occurred in developed countries following the burst of the dotcom bubble. During 2008–2010 global financial crisis, the government and central bank responded with unprecedented fiscal stimulus, monetary policy expansion, and institutional bailouts to stimulate the economy.

[7] For the quantitative easing, as $\ln g_t$ and $\ln g_{t-1}$ follow the same ergodic distribution, we have $\operatorname{var}(\ln g_t) = \frac{\chi^2}{1-\rho_\sigma^2}\operatorname{var}(\sigma_{g,t})$ and $E(\ln g_t) = \ln\bar{g}$. Similarly, $\ln\sigma_{g,t}$ and $\ln\sigma_{g,t-1}$ follow the same ergodic distribution, and we have $E(\ln\sigma_{g,t}) = \ln\bar{\sigma}_g$ and $\operatorname{var}(\ln\sigma_{g,t}) = \frac{\eta_\sigma^2}{1-\rho_\sigma^2}$. Therefore, $\sigma_{g,t}$ follows log normal distribution, $\ln\sigma_{g,t} \sim N\left(\ln\bar{\sigma}_g, \frac{\eta_\sigma^2}{1-\rho_\sigma^2}\right)$.

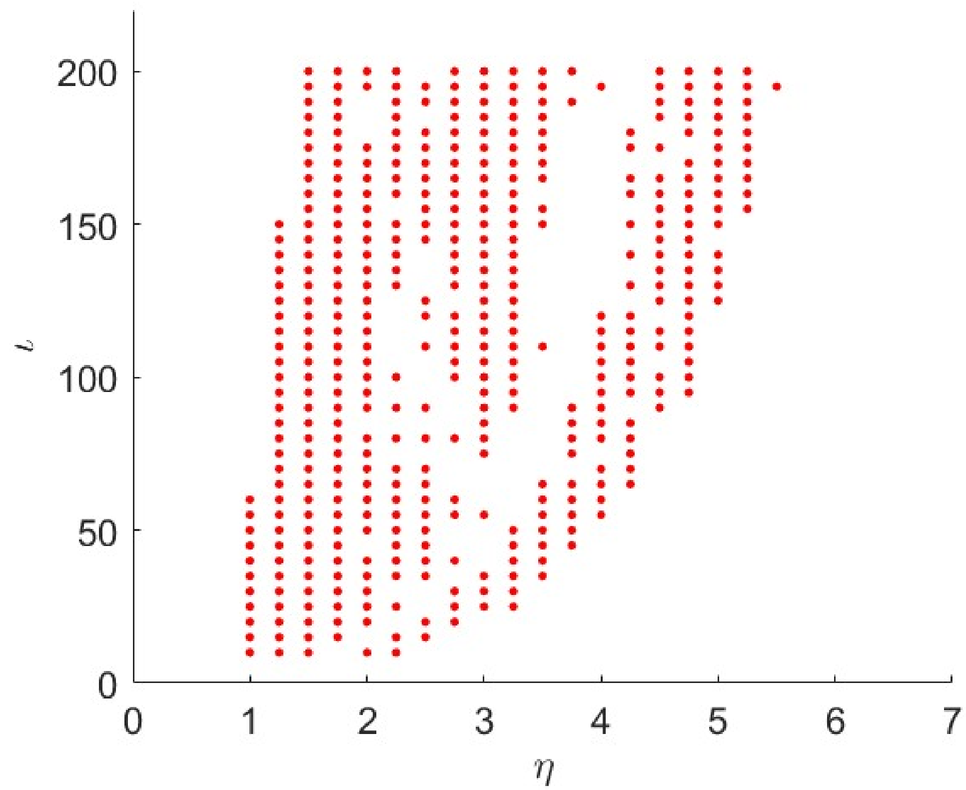

Figure 5: AR(1) model and growth rate of money supply

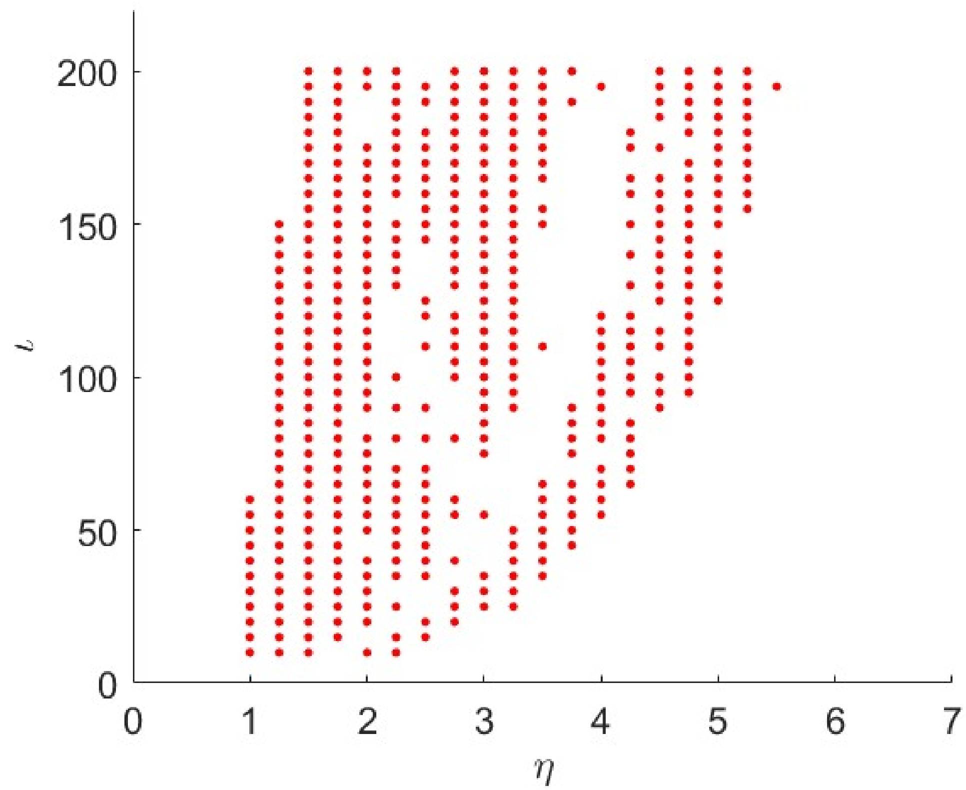

Figure 6: Residual plots of growth rate of money supply

Appendix D. Stochastic volatility of money growth rate

We estimate the time-varying volatility of money growth rate from the residual of lng_t using the following GARCH (1,1) model in Table 6:

$$\sigma^2_{g,t} = \alpha + \beta\sigma^2_{g,t-1} + \gamma\varepsilon^2_{g,t-1}. \tag{19}$$

Table 6: GARCH (1,1) estimate result

Parameter	Value	Standard Error
Constant	5.1086×10^{-6}	4.8103×10^{-6}
GARCH(1)	0.9	0.0476
ARCH(1)	0.05	0.0210

After the GARCH (1,1) model generates value of stochastic volatility of money growth rate at each time period, we estimate equation (6) in Table 7.

Table 7: Estimation of AR(1) volatility in money growth rate

Description	Parameter	Standard Error	p-value
Constant	-0.2609	0.1105	0.0192
AR(1) coefficient: ρ_g	0.9435	0.0240	$< 10^{-4}$
RMSE	0.0466		
R-square	0.888		

Based on the results, we set smoothing coefficients of ambiguity in forward guidance $\rho_\sigma = 0.9435$, indicating that stochastic volatility has an important role. Similarly, η_σ and $\bar{\sigma}_g$ can be determined. The value η_σ is the standard deviation of residual from this regression, $\eta_\sigma = 0.0466$, and the steady state ($\bar{\sigma}_g$) can be obtained using the equation $(1-\rho_\sigma)\ln\bar{\sigma}_g = -0.2609$, resulting in $\bar{\sigma}_g = 0.0099$.

The AR (1) coefficient is significant and model estimate matches the data well; therefore, it can explain the majority of the data series. Outliers occur in the early 1980s, early 2000s, and 2008–2010. All of these outliers can be explained by the real economy.

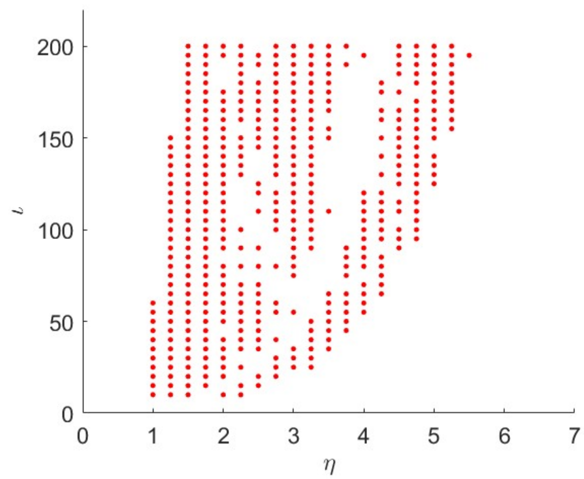

Figure 7: AR(1) model and volatility in money growth rate

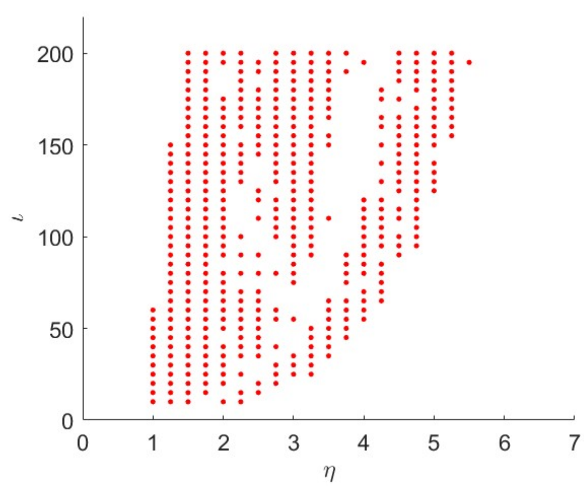

Figure 8: Residual plots of volatility in money growth rate