The cost of default in a cash-in-advance economy

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Abstract

The 2008 global financial crisis and its aftermath caused the high default rate and thus many national central banks ran expansionary monetary policy. Under a Cash-In-Advance (CIA) framework, we examine the welfare cost of default and how the injection of cash affects welfare cost of default. Incorporating non-linear form of non-pecuniary default penalty into a CIA model, we find that default causes around 1% loss in consumption. The negative effects of default on welfare is significantly greater in an economy with higher money growth rate only in the long-run but not in the short-run.

Keywords: Welfare cost; Default; Monetary Policy; Cash-In-Advance
JEL Classification: E44; E52; G01; I30

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1 Introduction

The recent financial crisis in 2008 caused a high level of default rate. In 2009, Moody's global speculative-grade default rate peaked 13 percent, and it was followed by a global economic downturn. For example, the U.S. personal consumption expenditure fell below 92 percent of disposable personal income as of 2009. To escape from the Great Recession in 2008, the US government implemented the expansionary monetary policy, known as "Quantitative Easing" during 2009 and 2010. It added around $2 trillion to the money supply. Although the coexistence of widespread default in the credit market and a large injection of cash was unprecedent in the U.S. financial crisis history, little studies have been done evaluating the effects of both default and monetary policy on social welfare.

As such, we estimate the welfare cost of default and investigate how the welfare cost of default varies with abnormal money injection. First, to assess the welfare cost of default, we build a Dynamic Stochastic General Equilibrium (DSGE) model incorporating both Cash-In-Advance (CIA) constraints and endogenous default. The welfare cost is measured as the percentage change in real consumption, to maintain the same level of utility with that in the benchmark economy without default (Lucas, 1987; Cooley and Hansen, 1989). Second, we measure default rate as the probability of not repaying at all, and construct default penalty as a non-linear form of utility loss. The default penalty increases non-linearly with the default volume and credit-regulation level. Third, to model the money supply rule, we use AR (1) process on money growth rates. This simplified monetary supply rule reflects the fact that the Federal Reserve Board implemented a policy by maneuvering quantities since interest rates were too low to drop (Adrian and Shin 2009).

Using the theoretical framework, we find that default reduces social welfare. Default risk reduces corporate loan volume, which leads to lower investment and output. As a result, households consume less. The presence of default causes social welfare loss by around 1% loss in consumption. Since firms bear the deadweight loss of default, stricter credit-regulation will lower default rate and improve social welfare at the cost of lower firm profits.

In addition, we find that the welfare cost of default increases with money growth rate, and this negative effect of the expansionary monetary policy on the welfare cost is nonnegligible in the long-run. Because the time is long enough to accumulate the propagation effect caused by default in credit market to significantly reduce the consumption. It is more social welfare costly for a CIA economy with higher money growth rate. However, in the short run, this propagation effect is small, and thus the welfare cost is not significant. This may lead to monetary policy inertia. The policy maker may not respond to the default behavior quickly enough if they haven't observed a significant welfare loss.

To our best of knowledge, this article is the first study that assesses the welfare cost of default by incorporating with monetary policy. Since the 2008 global financial crisis, many governments have implemented expansionary monetary policy to recover the busted economy. Thus, to incorporate the government monetary policy into a model provides the more comprehensive understanding the welfare cost of default. In literature, most research about cost of default emphasized the profit loss to individual firms (Warner, 1977; Altman, 1984; Weiss, 1990; Davydenko et al., 2012; Glover 2016). The welfare cost of default haven't been studied thoroughly, let alone how monetary policy affects welfare cost of default. The findings in our study help policymakers consider in which way they implement their monetary policy to cope with a financial crisis with high default rates. Furthermore, this study contributes to the literature of CIA models to overcome the limitation of standard CIA models. Standard CIA models are not feasible to generate the positive response of output to unexpected money growth shock. To overcome this limitation, we use a non-linear form of non-pecuniary default penalty. With the existence of non-linear default penalty, default rate is lower and lower default risk premium boosts lending in the corporate loan market and result in higher output.

The structure of this paper is as follows. Section 2 reviews past literature. Section 3 presents our model, which forms the basis for our analysis. Section 4 lays down equilibrium analysis such as market clearing conditions and optimality conditions. Section 5 explains calibration of the model. Section 6 discusses our results based on quantitative analysis, including cyclical properties, steady states and estimation of welfare costs of default in our economy. Sections 7 provides conclusions.

2 Literature review

This paper relates to several streams of studies in the literature.

2.1 Monetary models with CIA constraints

There's a wide agreement in the macroeconomics literature that nominal or monetary factors is important since Lucas (1972, 1973, 1975). To further theoretically study the role of money, Lucas (1982) and Lucas and Stokey (1987) initiated the "cash-in-advance models" where goods can be identified as distinctive "credit goods" or "cash goods" depending on timing and agents hold money due to "cash-in-advance constraints" (Clower, 1967) for buying "cash goods". The Clower type CIA constraint implies that agents must have enough cash balance before they can purchase "cash goods", and it captures the role of money as a medium of exchange. Following basic framework of Lucas and Stokey (1987) and Hansen (1985), Cooley and Hansen (1989) introduced money into the "Real Business Cycle model". It revealed the co-movements between real and nominal economic variables and claimed that the anticipated inflation tax induced by money supply growth causes huge social welfare loss. After that, CIA models became a popular approach to model money in dynamic macroeconomic framework because it is a simple way to model money demand and reflect any surprise in monetary policy.

But monetary RBC models with CIA constraints have some drawbacks in monetary policy effects analysis. According to Schorfheide (2000), monetary RBC models with CIA constraints fail to generate positive response of real output to an unanticipated positive shock in monetary policy. It suggests that in order to successfully account for the interaction between real and nominal variables, more sources of non-neutrality than just the inflation tax should be considered.

2.2 Financial frictions in macroeconomics

Financial frictions came into macro economists' view since Bernanke and Gertler (1989)'s overlapping-generations model. Bernanke and Gertler (1989) started from information asymmetry, to model financial friction. After that, there are mainly two approaches to introduce financial frictions into the general equilibrium framework.

The first approach is explored by Kiyotaki and Moore (1997) which is based on collateral constraints and emphasizes more on the quantity of loans. In their model, lenders are not allowed to force borrowers to fulfill their obligations, and borrowers cannot borrow money unless their debts are secured. Generally, durable assets are used as collateral for loans.

Financial markets have influence on real economic activities through interaction between durable asset prices and credit limitation. Many applications based on this framework appeared after that. Iacoviello (2005) introduced housing as collateral and more recently Gerali et al. (2010) assumed that the inter-bank financing ability of banks is limited by the asset portfolio available for collateral to study the impact of financial frictions on monetary policy transmission.

The other approach is a continue of Bernanke-Gertler agency-costs economy which accentuates the role of external finance premium. Carlstrom and Fuerst (1997) extended Bernanke-Gertler model to infinite time horizon and later Bernanke et al. (1999) merged this approach with the New-Keynesian framework, which became a classical reference for financial frictions modelling. In this type model, borrowers and lenders are different with conflict of interest, leading to agency problem. Monitoring the credit market is costly, and agency costs are constructed as endogenous. Financial markets affects business cycles mainly through the prices of loans in which credit risk premium is considered. Christiano et al. (2004), Christiano et al. (2007) and Christiano et al. (2008) further used the external finance premium setup to study the role of financial frictions in business cycles. In this paper, the external finance premium approach is adopted to study the role of default.

2.3 Default and cost of default

Among all possible financial frictions, Goodhart et al. (2006) and Tsomocos (2003) suggested to mainly concentrate on two factors, money and default, to study the monetary mechanism and financial stability. Shubik and Wilson (1977) first introduced the the possibility of default as equilibrium phenomenon that borrowers can choose the optimal fraction of debts to repay. Following this setup, Goodhart et al. (2006), Tsomocos (2003), and Tsomocos and Zicchino (2005) modelled endogenous default into the economy to study the role of default in monetary mechanism. Agents are allowed to choose their repayment rates, and thus equilibrium in such economy is compatible with complete or partial abrogation of borrowers' obligations. If it is not worthy to repay any of their debts, agents will rationally default all.

Then introduction of the cost of default is important. To fully define all possible outcomes in strategic market games, Shapley and Shubik (1977) introduced minimal institutions such as money, credit and default, and the rules of default penalties to be levied against agents who choose to default. Geanakoplos (1997) proposed to model penalties of default on secured debts as pecuniary penalty based on the loss of collateral, which at least guarantees partial payment; while default on unsecured debts are constructed as pecuniary loss or non-pecuniary loss. The pecuniary penalties can be considered as garnishing of future income or a search cost for new financial resources, and the non-pecuniary default can be considered as pangs of conscience, a prison term or reputation loss. In the field of non-pecuniary default penalty, Tsomocos (2003) is one of the papers that pioneer to incorporate default penalties on borrowers into general equilibrium framework and constructed the cost of default as a default penalty that reduces the borrowers' utilities. Dubey et al. (2005) imposed a default penalty in the form of reputation loss in a multi-period world which will make agents keep their promises to pay back their debts. The default firms fear to lose new financing opportunities in the future. In addition, according to Dubey et. al (2005), the default penalty generally increases with the nominal amount of default.

Until now, there are very few articles that study the welfare cost of default and the welfare consequences are ambiguous according to the literature. In Dubey et al. (2000)

and Dubey et al. (2005) default causes social benefit in the incomplete market setting by enlarging asset span. While in Barro (1976) default causes net social welfare loss due to the asymmetric information between the borrower and lender. In their setting, default on a loan triggers the loss of collateral value to the borrower, making the lender valuation much below borrower valuation during the negotiation.

The finding of this paper is consistent with the second stream of the literature (e.g., Barro 1976) that default on a loan causes net social welfare loss and it contributes to the literature by estimating the magnitude of the welfare consequence.

3 The model

This section describes the established simple dynamic stochastic general equilibrium (DSGE) model with infinite time horizon. This model is extracted and extended from the CIA monetary DSGE model in Schorfheide (2000). In this economy, there are two agents, representative households and firms. Each household is identical, and so is each firm. The representative household and firm live forever. Households are endowed with labor, inherit the entire money stock of the economy from last period, and obtain utility from enjoying leisure and consumption. Firms use labor and capital to produce and sell final goods, and obtain utility from profits. Firms are owned by households, and thus in the end of each period, firms distribute all their profits to households in the form of dividends.

Households face CIA constraints. Before they purchase consumption goods, they should have enough money (cash) in their hands. So consumption goods is "cash goods" in this economy. The setup of CIA constraints enable the economy completely reflect the current period's surprise in technology and money supply growth. Money is fiat and thus holding idle money in hand does not generate any utility for agents. They will lend all the idle money out to someone who need it. Figure 1 presents the diagram of the model economy. Households and firms play their roles in goods market, credit market, labor market and corporate loan market. The nominal price level, P_t, is subject to the interplay of demand and supply in the final goods market.

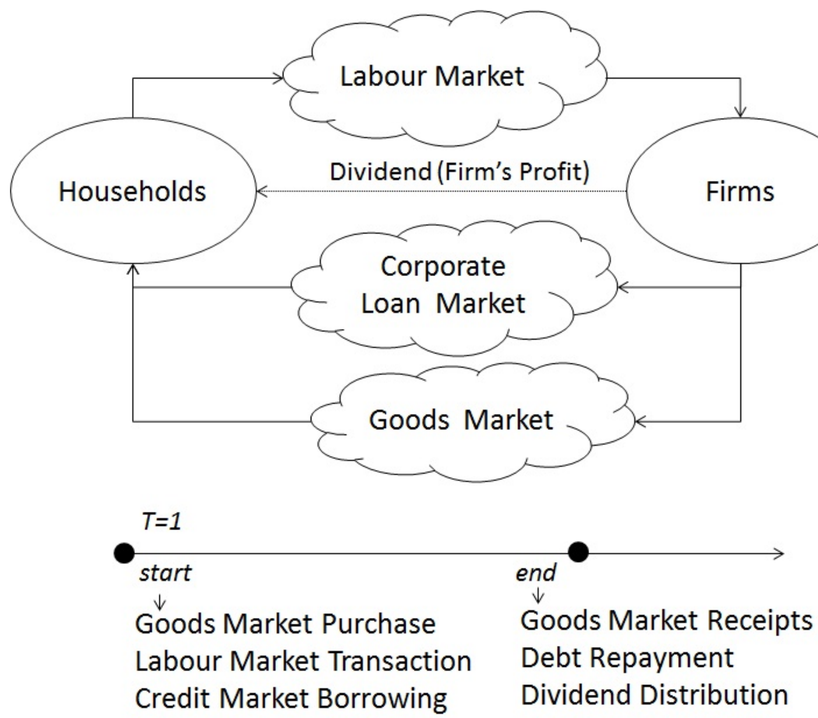

Figure 1: The model economy

At the beginning of period t, with the inherited money, households decide how much they will use to purchase final goods for current period consumption, how much they will lend to firms at the interest rate R_t and how much they will keep for next period economic activities.

Since firms distribute all their profits to households in the end of each period, firms start each period with no remaining money and thus they have to borrow money from households via corporate loan market to hire workers. They sign labor contracts with households and pay wages by using the money they finance from corporate loan market. With labor and capital inputs, firms produce and sell final goods through goods market. In each period, firms should also decide how many produced goods they will use as investment for next period production.

To introduce default, firms are allowed to default their debts if they think fully repay all their debts are not worthy and they can choose optimal repayment rates to maximize their utilities. If the firm defaults, it bears non-pecuniary penalty which shows up in it's utility function and increases with the real size of default exponentially.

Households

Households work, consume and lend money to firms. In each period, the representative household chooses the optimal quantity of goods for consumption C_t, labor supply H_t, cash holding for next period m_{t+1} and investment in corporate loan market (the amount of money lend to firms) D_t to maximize the expected sum of discounted lifetime utility. As a result, it solves the optimization problem as follows,

$$\max_{\{C_t,H_t,m_{t+1},D_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{(1-\phi)\ln C_t + \phi \ln(1-H_t)\right\}$$

$$s.t.$$

$$P_t C_t \le m_t - D_t + W_t H_t + (g_t - 1) M_t \tag{1}$$

$$0 \le D_t \tag{2}$$

$$m_{t+1} = (m_t - D_t + W_t H_t + (g_t - 1) M_t - P_t C_t) + \nu_t R_t D_t + F_t \tag{3}$$

where $E_0(\cdot)$ is the expectation operator conditional on date 0, β refers to the discount factor, ϕ is the preference shifter which reflects the marginal rate of substitution between leisure and consumption, ν_t is the repayment rate determined by the firm, W_t is the nominal wage, F_t is the dividend coming from firms, M_t denotes the per capita money supply in period $t-1$, and the money stock in this economy follows a law of motion $M_{t+1} = g_t M_t$.

The first constraint the household faces is the CIA constraint. The household has money inflow from last period money stock inheritance m_t, current period money injection $(g_t - 1) M_t$, wage revenues $W_t H_t$. At the same time, s/he lends money D_t out to firms and purchases consumption goods with expenditure $P_t C_t$. The money used for consumption cannot exceed the household's money balance $m_t - D_t + W_t H_t + (g_t - 1) M_t$.

The second constraint means the representative household cannot borrow money from firms. Households have no debts outstanding.

The third constraint is the inter-temporal budget. Other than the remaining money balance after consumption, the household gets money from the firm in the end of each period. As a creditor, the household gets repayment with the amount of $\nu_t R_t D_t$ from the firm; as a owner, the household received cash dividend distribution from the firm with the amount of F_t.

Firms

Firms borrow money from households, hire workers and produce goods. They issue corporate bond to hire workers in the beginning of each periods, and pay back optimal fraction of their debts outstanding in the end of each period. Specially firms are allowed to default at the cost of non-pecuniary penalty. To balance the benefits and costs of the default, firms will choose their optimal repayment rates.

Firms use labor and capital to produce goods. Final goods are produced following a "Cobb-Douglas production function",

$$Y_t = K_t^{\alpha} (A_t N_t)^{1-\alpha} \tag{4}$$

where K_t denotes the capital stock which is predetermined at the beginning of period t, A_t is a labor-augmenting technology and N_t represents the labor input. α and $(1-\alpha)$ represent output elasticity of capital and output elasticity of labor respectively.

Capital is accumulated according to the law of motion $K_{t+1} = (1-\delta)K_t + I_t \ (0 < \delta < 1)$, where I_t is the new investment in current period.

In each period, the representative firm should make optimal decisions on labor demand N_t, bond issued L_t, repayment rate ν_t, nominal cash dividends F_t, and the next period's capital stock K_{t+1}. Since households receive the dividend in the end of each period, and thus value a unit of nominal cash dividend in terms of purchase power in next period. Firms distribute all their profits to households in the end of each period, so the current period nominal dividends are discounted by marginal utility of consumption in next period. Moreover, firms care about the default penalties to be levied on them.

To be brief, the representative firm faces the optimization problem as follows,

$$\max_{\{F_t, K_{t+1}, N_t, L_t, \nu_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left\{ \frac{F_t}{C_{t+1} P_{t+1}} - \frac{\iota}{1+\eta} \left[(1-\nu_t) R_t \frac{L_t}{M_t} \right]^{1+\eta} \right\}$$

$$s.t. \qquad W_t N_t \leq L_t \tag{5}$$

$$F_t = L_t + P_t \left[K_t^{\alpha} (A_t N_t)^{1-\alpha} - K_{t+1} + (1-\delta) K_t \right] - W_t N_t - \nu_t R_t L_t \tag{6}$$

where ι and η denote the coefficient and the elasticity of non-pecuniary default for firms.

When ι is higher, the regulation in the credit market is stricter and thus firms hesitate more when they make a decision to default. When η is higher, the default penalty increases more quickly with the real size of default, and the firm cares more about negative effects of default. To some extent, ι can be explained as the degree of default regulation and η can be explained as the firm's degree of risk-aversion.

The first constraint shows that the firm borrow money from households through corporate bond to pay wages to workers at the beginning of each period.

The second constraint shows that the firm obtains the sales revenue, repays $\nu_t R_t L_t$ to households, invests more for next period production and distributes the remaining cash to households as the dividend.

Note that since money supply growth rate is greater than discount factor, CIA constraints and budget constraints for both households and firms are binding.

Technology is assumed to follow a stationary AR(1) process,

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + (1-\rho_A) \ln \bar{A} + \sigma_A \varepsilon_{A,t} \\ \varepsilon_{A,t} &\sim i.i.d.\, N(0,1) \end{aligned} \tag{7}$$

ρ_A refers to the AR(1) coefficient of technology and $\bar{A}$ indicates the steady state of technology. The innovation $\varepsilon_{A,t}$ follows a $N(0,1)$ process where σ_A denotes the standard deviation of innovations to $\ln A_t$.

Monetary policy

The money growth rate in current period is defined as $g_t = M_{t+1}/M_t$, where M_t is the money supply in period $t-1$. In each period, the central bank injects $(g_t - 1) M_t$ money into this economy. To make our analysis simple and straightforward, it is assumed that the inter-temporal money growth rate is stable in the long run and the log-linearized forms of g_t follows a stochastic AR(1) process,

$$\begin{aligned} \ln g_t &= \rho_g \ln g_{t-1} + (1-\rho_g) \ln \bar{g} + \sigma_g \varepsilon_{g,t} \\ \varepsilon_{g,t} &\sim i.i.d.\, N(0,1) \end{aligned} \tag{8}$$

Where $\bar{g}$ is unconditional mean, $\varepsilon_{g,t}$ is the innovations to the monetary policy which follows a standard normal process, and σ_g measures the standard deviation of monetary innovations. Thus, the last term $\sigma_g \varepsilon_{g,t}$ captures the unexpected monetary policy shock due to sudden nominal policy making.

Equation (8) can be interpreted as a simple monetary policy rule without feedback. The monetary policy shock is modeled as exogenous to investigate the transmission mechanism.

4 Equilibrium analysis

4.1 Market clearing conditions

There are three markets in this economy: labor market, corporate loan market and final goods market. It is assumed that all markets are competitive. Thus, in the model economies, an equilibrium requires clearing all three markets.

For the labor market, the market clears when the supply of labor from households is equal to the demand of the labor from firms. Thus, $H_t = N_t$ holds for $\forall t \in T$.

For the corporate loan market, the amount of money that households are willing to lend is equal to the amount of money that firms need for paying wages. Thus, $D_t = L_t$ holds for $\forall t \in T$.

Note that in equilibrium, the money supply equals to the money demand, $m_t = M_t$.

For the final goods market, the market clearing condition is that the demand for final goods from consumption and investment equals to the supply of final goods produced by firms. Thus, $C_t + I_t = Y_t$ holds for $\forall t \in T$. This market clearing condition can be derived from the budget constraint of households (3) and the budget constraint of firms (6), along with labor and money market equilibrium conditions.

Recall all budget the constraints of households and firms are binding. In an equilibrium, when we combine budget constraints (1) and (5),

$$P_t C_t = m_t - L_t + W_t N_t + (g_t - 1) M_t$$
$$W_t N_t = L_t$$

In equilibrium, money supply is epitomized, thus $M_t = m_t$. Then it further implies the equilibrium condition in the money market,

$$P_t C_t = M_{t+1} \tag{9}$$

According to the equation (9), the money demand which is denoted by nominal consumption demand $P_t C_t$ should be equal to money supply M_{t+1}, which can be represented by current nominal balances m_t and monetary injections $(g_t - 1) M_t$.

4.2 Optimality conditions

This part shows the results of optimality conditions derived from the optimization problems of households and firms face to give us some characteristics of this cash-in-advance economy with default.

According to the optimality condition for money market, $P_t C_t = M_{t+1}$, it shows that in the long run money is neutral in this model economy, which is in line with the argument in real business cycle (RBC) theory and New Keynesian literature. However, in the short run, money is not neutral in this artificial economy, which is in contrast with the standard RBC theory.

One thing should be clarified here is that the mechanism leading to short run money non-neutrality in this artificial cash-in-advance economy differs from that in New Keynesian framework. In this economy, money non-neutrality is driven by transaction demands for money and the financing channel where money and default have great influence; while under New Keynesian framework, real frictions such as monopolistic or asymmetric information are important.

According to the first order conditions (FOCs) derived from optimization problem of households, we can easily get following equations:

$$\frac{\phi C_t P_t}{(1-\phi)(1-H_t)} - W_t = 0 \tag{10}$$

$$E_t\left(\frac{\beta \nu_t R_t}{C_{t+1}P_{t+1}}\right) - \frac{1}{C_t P_t} = 0 \tag{11}$$

The optimality condition (10) shows that the marginal rate of substitution (MRS) between consumption and leisure must be equal to the nominal wage rate. The optimality condition (11) represents the stochastic consumption Euler equation: the marginal utility of consumption in current period should be equal to the expected weighted marginal utility of consumption in next period. In other words, it means if they do not consume today and lend out the money, the marginal utility of consumption today is equivalent to the discounted marginal utility of next period consumption enabled by the money received from repayment.

From this model, it also shows that the labor market optimality condition are closely associated with the credit market and households' consumption decision. One possible explanation is that firms have to borrow money through credit market (corporate loan market) to pay wage bills. So their financing ability will affect the demand side of the labor market. In detail, the optimality condition can be derived from combining the firm's borrowing constraint $L_t = W_t N_t$, labor market equilibrium $H_t = N_t$ and $D_t = L_t$ and FOCs,

$$\frac{\phi C_t P_t}{(1-\phi)(1-N_t)} - \frac{L_t}{N_t} \quad = \quad 0 \tag{12}$$

Equation (12) suggests that when the ratio of money invest in corporate loan market and expenditure used in consumption goods purchase increases, there will be more workers employed in the labor market.

For the maximization problem of firms, we get following optimality conditions by combing FOCs and equilibrium conditions:

$$E_t\left[\frac{P_t}{C_{t+1}P_{t+1}} - \frac{\beta P_{t+1}}{C_{t+2}P_{t+2}}\left(\alpha K_{t+1}^{\alpha-1}\left(A_{t+1}N_{t+1}\right)^{1-\alpha} + (1-\delta)\right)\right] = 0 \tag{13}$$

$$\frac{(1-\alpha)P_t Y_t}{L_t} - R_t = 0 \tag{14}$$

$$\frac{\iota\left[(1-\nu_t)R_t L_t\right]^{\eta}}{M_t^{1+\eta}} - E_t\left(\frac{1}{C_{t+1}P_{t+1}}\right) = 0 \tag{15}$$

The first condition (13) is the optimality condition for investment and consumption. It tells the the trade-off the economy faces: invest more by delaying the consumption to next time, or consume today.

The second condition (14) tells that the equilibrium interest rate is determined by the borrowing of firms and consumption decisions of households. Firms make the nominal sales revenue causing by labor inputs equal to the nominal cost of labor which is denoted by debts and financing cost for wage bills.

The third condition (15) identifies how firms make the optimal default decision. There is pros and cons to default. On the one hand, when firms default, they escape from their obligations and thus have more cash in their pockets. Firms will distribute more cash dividend to households, thus in turn enables households to consume more. On the other hand, the default firms have to suffer certain amount of non-pecuniary default penalty which non-linearly increases with the real amount of default.

Proposition 1: Fisher Effect

For the artificial economy, suppose that economy works in both goods and corporate loan markets, i.e. the representative household's consumption $C_t > 0$, the repayment rate $0 \leq \nu_t \leq 1$ for $\forall t \in T$. Then we can get the following short-run equilibrium condition by taking the logarithm for Euler equations (11) ,

$$\ln R_t = \ln E_t \left(\frac{U'_{C_t}}{\beta U'_{C_{t+1}}} \right) + \ln E_t (\pi_{t+1}) + \ln \frac{1}{\nu_t}$$

It says that logarithmic form of the nominal interest rate is equal to the real interest rate plus inflation and the risk premium. Note that other than inflation risk, default risk is considered in this model. The "Fisher Effect" proposition identifies there key factors that are closely linked to the nominal variables: consumption, inflation and default risk premium. It also shows that when the nominal economic variables are affected, real variables will also be affected with regard to allocation.

Proposition 2: Quantity Theory of Money

In the model, the FOCs, binding CIA constraints and budget constraints imply the following equilibrium condition,

$$P_t C_t = M_{t+1}$$

"Quantity Theory of Money" proposition verifies the long run money neutrality. In the long time, the economy converges to steady state where $\frac{PY}{M}$ is constant.

However, in dynamic, we have the following condition derived from CIA constraints of two agents and goods market clearing conditions,

$$\frac{P_t Y_t}{M_{t+1}} = 1 + \frac{P_t I_t}{M_{t+1}}$$

It says that the investment decision is influenced by monetary policy and also this influence is transmitted into real economic activities. The nominal or monetary factors have significant effects on real economic activities. Then it confirms the non-trivial role of money in business cycles. In the short run, the "Quantity Theory of Money" proposition does not hold and money is not neutral in this economy.

Corollary 1: Money Non-neutrality

Since money is fiat, agents hold money balances at the cost of foregone interest. To make corporate loan market work, interest rate is assigned to be positive which reduce the efficiency of trade and transaction. Thus, monetary policy is non-neutral.

Proposition 3: On-the-Verge Conditions

In any equilibrium of the artificial economy, the detrended form * of the equations can be derived from the optimality condition (11) for $\forall t \in T$,

*See Appendix A.

$$E_t\left(\frac{1}{C_{t+1}\hat{P}_{t+1}g_t}\right) \quad = \quad \iota\left[(1-\nu_t)R_t\hat{L}_t\right]^{\eta}$$

The "On-the-verge condition" implies that the firm chooses the the optimal repayment rate based on the utility and the dis-utility of default. The behavior of default is double-edged. On the one hand, default enables the firm to escape from certain fraction of debts and distribute more cash dividend to households, and as a result, household can consume more. On the other hand, default make the firm under stress and bear reputation loss. The non-pecuniary default penalty will reduce the utility of the firm. According to this obtained condition, the firm will increase the repayment rate until the marginal utility of default equals to the marginal dis-utility of default.

4.3 Default and punishment

In standard general equilibrium framework, all agents keep their promises and repay all their debts outstanding. This paper incorporates agency problem of broken promises in to a dynamic general equilibrium model. In a economy where default is allowed, borrowers make repayment decision based on the trade-off between potential benefit and cost of default. This section discusses about the introduction of default and construction of default penalty in detail.

As mentioned before, Geanakoplos (1997) constructed default on secured debt as the loss of collateral, while generally default on unsecured debt induces pecuniary penalties or non-pecuniary penalties. This paper follows Tsomocos (2003) to incorporate endogenous default into dynamic general equilibrium framework. Agents can determine their credit default behavior. Furthermore, this paper adopts non-pecuniary form which shows up in borrowers' utility function initiated by Shubik (1973) and Shubik and Wilson (1977). As mentioned in Dubey et al. (2005), the non-pecuniary default can be considered as a reputation loss. If borrowers default their current debs, it is harder for them to obtain new loans in the future. Default penalties in Tsomocos (2003) and Dubey et al. (2005) are modelled by subtracting a linear term from the objective funciton of the default agents. In Dubey et al. (2005), default penalty is proportional to the nominal size of the debt. But in Tsomocos (2003), the nominal value of default are deflated by the money stock in the whole economy. This paper follows the latter form but differently, default penalty is in non-linear form.

To begin with, it assumes that the default penalty is increase with "real' amount of default (Tsomocos, 2003) and the non-pecuniary default penalty can be described as $MU_{\hat{l}t}(1-\nu_t)R_t\hat{L}_t$, where $(1-\nu_t)R_t\hat{L}_t$ is the "real" value of default and $MU_{\hat{l}t}$ is the marginal dis-utility of each 'real' dollar default on debts at time t. Following Dubey et al. (2005), default penalties are levied on borrowers who fail to keep their promises regardless of the reasons. So defaulters who fail to deliver due to ill fortune acquired the same amount of penalty as those who default due to fraud if their default levels are the same.

Moreover, the marginal dis-utility of each "real" dollar default on debts varies with time and it assumes that $MU_{\hat{l}t}$ is a function of the "real" amount of default. In this model, $MU_{\hat{l}t} = \frac{\iota}{1+\eta}\left[(1-\nu_t)R_t\hat{L}_t\right]^{\eta}$ for firms where ι is the constant coefficient of default penalty. Thus, the default penalty is constructed as $\frac{\iota}{1+\eta}\left[(1-\nu_t)R_t\hat{L}_t\right]^{1+\eta}$ for borrowers in this paper. With such a form of default penalty, the marginal dis-utility of each "real" dollar default is increasing with the "real" level of default. So the punishment of default is severer when the "real" level of default is higher.

In sum, this paper model the default as an equilibrium phenomenon and the default penalty in non-pecuniary and non-linear form following Shubik and Wilson (1977), Tsomocos (2003) and Dubey et al. (2005). Following Tsomocos (2003), this study models the default penalty which is associated with "real" value of default, but differently marginal default penalty is constructed as $MU_{\hat{l}t} = \frac{\iota}{1+\eta}\left[(1-\nu_t)R_t\hat{L}_t\right]^{\eta}$.

Moreover, by setting appropriate default penalty parameters (ι, η), this paper successfully addresses the limitation of standard CIA models in generating positive response of output in the case of positive monetary shock.

5 Calibration

This part discusses about the calibration for this model. The values of parameters in this model are chosen based on estimation from actual data of the U.S. and standard literature.

5.1 Data description

The data is mainly collected from FRED (Federal Reserve Economic Data) in quarterly frequency. Thus one period in this economy is one quarter. The sample period is from 1981Q4 to 2016Q4. For each series, there are 141 observations.

The U.S. time series reported on are Output, Consumption, Investment, Depreciation, Capital Stock, Labor, CPI, GNP Defaulter, Productivity and money growth rate. According to standard literature, the data sources of Output, GNP deflator, Consumption, Investment and Depreciation are chose as "Real Gross National Product", "Gross National Product: Implicit Price Deflator ", "Real Personal Consumption Expenditures ", "Real Gross Private Domestic Investment" and "Real Consumption of Fixed Capital: Private" from U.S. Bureau of Economic Analysis respectively. The labor comes from "Index of Aggregate Weekly Hours: Production and Nonsupervisory Employees: Total Private Industries" of U.S. Bureau of Labor Statistics. As for another indicator of price level, CPI comes from Organization for Economic Co-operation and Development (OECD) database. The money supply data is represented by the U.S. M1 which comes from OECD database. All series are seasonally adjusted.

Following the online appendix of Jermann and Quadrini (2009), the capital stock is constructed by using the equation,

$$K_{t+1} = K_t - Depreciation + Investment$$

We start the recursion in the first quarter of 1950 and drop the values before 1981Q4. Since the initial value of K_t is important only for the early steady states, this is not relevant for our results based on the sub-period 1981Q4-2016Q4.

Productivity calculated by the production function $Y_t = K_t^{\alpha}(A_tN_t)^{1-\alpha}$. In detail, initial labor-augmenting productivity factor can be calculated by

$$\ln(A_t) = \frac{\ln(Y_t) - \alpha \ln(K_t)}{1-\alpha} - \ln(N_t).$$

5.2 Parameters

This paper choose parameters mainly based on standard macroeconomics literature. The discount factor in this economy is chose as $\beta = 0.99$, a default choice. In order to get an reasonable capital-output ratio, this paper follows Fernández-Villaverde (2010) to set depreciation rate as $\delta = 0.025$, and follows Schmitt-Grohé and Uribe (2003) to set the U.S. capital share as $\alpha = 0.32$ in the "Cobb-Douglas production" function. Following Nason and Cogley (1994), the preference shifter which reflects marginal rate of substitution between labor and consumption is $\phi = 0.773$ in this economy. As for the ι, we use reverse engineering to determine its value by fixing $\bar{\nu}$.

The parameters for stochastic process of labor-augmenting technology and money growth rate are estimated from simple regression using actual data (see Appendix). The AR(1) coefficient of labor-augmenting technology ρ_A is 0.8063 and the standard deviation of technology innovation σ_A is 0.0066. To simplify the analysis, the value of labor-augmenting technology in the steady state is normalize as $\bar{A} = 1$. As for AR(1) parameters of the money growth rate, the AR(1) coefficient is $\rho_g = 0.3522$, the standard deviation of monetary policy innovation is $\sigma_g = 0.0168$ and unconditional mean $\bar{g}$ is 1.0147. It also implies that the growth rate of money injection in steady state is 1.0147.

Thus, all the calibrated values for parameters are well within the range of values in standard macroeconomics literature or comes from the features of the actual U.S. data. Table 1 reports all of these implied parameters.

⟨Insert table 1 here⟩

6 Quantitative analysis

In this section, the artificial economy mentioned above is used to study the interaction between financial frictions and the real sector of the economy. This paper first describes cyclical properties of the CIA economy with and without default under various conditions. Then, this paper investigates how the endogenous variables of the model respond to technology and monetary shocks in the economy with default. Furthermore, this paper uses the model to evaluate the impact of regulatory policy on default and measure the welfare costs of default. Finally, it confirms the implied steady-state behavior of economies with various default rates.

6.1 Cyclical properties

This section mainly explains and summarizes the statistics with and without default and study the impact of temporary technology and monetary shocks. The goal of this section is to check the cyclical properties of the economy and the transmission mechanism of the technology and monetary shocks in a CIA economy with default.

The analysis starts from check the statistics of the economic variables including output, consumption, investment, capital stock, labor, productivity and price level. In Table 2, we study the performance of the recent U.S. economy, the CIA economy with default, and the CIA economy without default. The standard deviations of the main economic variables and the correlations with the real output are listed in Table 2.

When study the U.S. economy, the sample period is from 1981Q4 to 2016Q4 and data is in quarterly frequency. When study the artificial CIA economy with default, the results comes from 50 times simulations. In each simulation, there are 141 periods, in line with actual data. When do the simulation, this paper simulates 241 periods and drops the first 100 periods. In panel 2 and 3 of Table 2, the listed standard deviations and correlations with the real output comes from taking the average of the results of 50 times simulations.

The sample standard deviations of the results from simulation are also listed in parentheses. Both the actual and simulated time series of the economic variables are logged and detrended by using the HP (Hodrick-Prescott) filter before calculating the statistics.

Comparing the statistics of two artificial economies, it shows that the incorporation of default does not affect the features of the business cycle. The signs and magnitudes are almost the same, consistent with the results of actual data. The sample standard deviations of simulated results in the CIA economy with default and without default look similar. It indicates that default does not change the business cycle features of an economy.

⟨Insert table 2 here⟩

Then, this paper further checks the IRFs of temporary technological innovation in the following part to study the role of default and cyclical properties of main Economic variables.

Positive technology shock

Since technology is treated as the most important factor causing economic fluctuations, we examine the IRFs of key macro-variables with respect to positive productivity shock in the CIA economy with default. The results are reported in Figure 1.

Firstly, in the labor market, it generates a positive response of wage rate and labor supply. A high level of productivity improves the marginal product of labor, thus the wage rate increases. With the higher wage rate, there are two competing effects. One is the substitution effect; the higher wage rate increases the opportunity cost of leisure and encourages people to work more, driving labor inputs up. The other is the income effect; with a higher wage rate, households want to enjoy more leisure. Substitution effect dominates income effect and results in higher income of households.

Secondly, in goods market, the consumption increases and the price level decreases. To begin with, with higher income, households want to consume more. Even though the demand effect pushes the price level up, positive technology shock directly increases output and the strong positive supply effect finally induces the price level to go down. Moreover, people seek to smooth consumption over time, which implies that increases in output and consumption will result in an increase in investment and therefore the capital stock. After that, increased capital stock and labor used for production and higher investment of firms all contribute to improve output again.

Thirdly, in the corporate loan market, the corporate loan increases and interest rate decreases. In contrast with the case of no default, the interest rate responds strongly to the technology shock. This can be explained by 'On-the-Verge Conditions' (Proposition 3) and 'Fisher Effect' (Proposition 1). According to the analysis of goods market, households consume more in the following several periods and the marginal utility of consumption in the next period goes down. In addition, according to the analysis of the labor market, firms have to borrow more to pay wages due to higher labor cost. The demand for corporate loan increases and the pro-cyclical property of corporate loans is confirmed. For 'On-the-Verge Conditions' (Proposition 3) to hold, with lower marginal utility of future consumption and higher corporate loan, the repayment rate increases. It generates a counter-cyclical risk premium. As a result, lower risk premium and marginal utility of consumption lead to lower interest rate based on 'Fisher Effect' (Proposition 1). It further stimulates investment and consumption.

It shows that the endogenous repayment rates generate a counter-cyclical risk premium. According to 'Fisher Effect' (Proposition 1), the interest rate decreases due to lower marginal utility of consumption and risk premium. Thus the demand for corporate loan increases and investment investment increases. Firms want to hire more workers. Compared with the economy without default, the wage rate is higher.

Positive monetary shock

We investigate the effects of expansionary monetary policy, as illustrated in the impulse responses to a money growth shock reported in Figure 3. We can observe that money growth shock has real effects on output and consumption due to CIA constraint and financial frictions. Thus, the non-trivial role of money and default is confirmed.

Recall that in standard CIA models, the output negatively respond to the positive monetary shock. But in this modified artificial economy, it successfully generates plausible response of output to the monetary policy change.

To begin with, abnormal money injection brings inflation and higher price level. The more inflation there is, the less money people would like to hold. Because inflation can be considered as a tax on the holders of money. However, since in equilibrium they cannot hold less money than the central bank prints, people try to get away from money (consumption) and into leisure, so consumption and employment all go down immediately. This effect lasts for several periods.

Since the labor supply decreases, the wage rate in corporate loan market increases, even more than the percentage decrease of employment. Firms have to borrow more in corporate loan market to pay the wages. The nominal interest rate increases because the nominal interest rate is approximately equal to the real interest rate plus expected inflation ("Fisher Effect", Proposition 1).

The repayment rate in this economy increases suddenly. Even though consumption decrease, the price level increases more. The utility of default decreases. While at the same time, with higher nominal interest rate and real size of corporate loan, the dis-utility of default tends to go up. For 'On-the-Verge Conditions' (Proposition 3) to hold, the repayment rate increases.

When the repayment rate increases, the real interest rate decreases ("Fisher Effect", Proposition 1). The investment goes up and it pushes up the output. If the elasticity of default penalty is big enough, the positive effects on the output from higher repayment rate can dominate the negative effects on the output from lower labor supply. In this artificial economy, when firm's default penalty parameters (η, ι) are in the parameter zone of figure 2 (in red), it can generate positive response of output as illustrated in Figure 4. In the calculation of steady state and dynamics, $\eta = 2$ and $\iota = 150$.

6.2 Welfare costs of default

In this section, the welfare costs of default are presented by comparing steady states of models with varies default levels. Cooley and Hansen (1989) take similar approach to measure the cost of the inflation tax. To study the short-run welfare effect, we also follow Lucas (1987) and calculate the welfare cost by simulating the dynamic.

6.2.1 Welfare cost of default: long run effect

Since as shown above the cyclical characteristics of this economy are unaffected by the default, we measure the welfare costs in the long run by comparing steady states. In detail, the welfare measure adopted is based on the increase in consumption that the representative household would require in the economy with default to be as well off as in the basic CIA economy without default. First, we get $\triangle C$ from the following equation for further welfare calculation,

$$\bar{U} = [(1-\phi)\ln(C^* + \triangle C) + \phi \ln(1 - H_t^*)]$$

where $\bar{U}$ is the maximized utility level in the steady state of the economy without default, while C^* and H^* are optimal consumption and labor (working time) in the steady state of the economy with default.

$\triangle C$ here can be considered as consumption compensation (loss) for the representative household in the economy with default. If $\triangle C$ is positive, households in the economy with default should be compensated with positive consumption to be as well off as in the economy without default, otherwise they are worse off. Then default option causes welfare loss. If $\triangle C$ is negative, households in the economy with default can be as well off as in the economy without default even if they consume less than the current optimal consumption level. Then default option brings welfare improvement.

Furthermore, to consider the size of the economy, this paper follows Cooley and Hansen (1989) to take the ratio of consumption compensation to steady-state real consumption ($\triangle C/C$) as the measure of the welfare cost.

Then we simulates three scenarios associated with varies money growth rates: low inflation economy ($g = 0.9965$), mediate inflation economy ($g = 1.0149$) and high inflation economy ($g = 1.0333$). The three different values of money growth rate are chose based on the basic statistics of actual U.S. M1 data (See the footnote of Table 3). In each scenario, this study sketches three sub-scenarios stretching from variation of default level: (1) low level of default ($v = 0.9941$); (2) mediate level of default ($v = 0.9968$); (3) high level of default ($\nu = 0.9906$).

⟨Insert table 3 here⟩

The results of welfare analysis are shown in the bottom rows of each panel of Table 4. Repayment rates are in quarterly frequency.

⟨Insert table 4 here⟩

Our estimate of the welfare cost of default is around 0.96% to 1.65% when $g = 1.0149$, which will be even higher if $g = 1.024$. While an estimate of roughly 1% of consumption sounds small, at the consumption level of 2017Q2 it would amount to 118.397 billions of chained 2009 dollars. Obviously this welfare loss due to default is non-negligible, comparing to the welfare cost of inflation estimated by Cooley and Hansen (1989) which is 0.52% of the consumption level when $g = 1.024$. This result emphasizes the important role of default in an economy.

As we can see, welfare costs increase with the severity of default in this artificial economy, which can also be confirmed by the results listed in panel 2 and 3 of Table 4. It can be explained as follows: on the one hand, the decreased default penalty lowers marginal utility of consumption and thus pushes up consumption in optimal allocation; on the other hand, firms will default more when they face loose regulation. According to "Fisher Effect" (Proposition 1), higher default rate requires higher risk premium which pushes up interest rate in corporate loan market. With higher financing cost, firms demand less corporate loan and thus reduce their investment and production, and it results in lower level of consumption. Higher marginal utility of consumption in turn pulls up default rate through "On-the-Verge

Conditions" (Proposition 3), which amplifies the effect of default penalty. In the end, the second effect dominates the first one.

Furthermore, it concludes that at the same default level, the cost of default is higher in the economy with higher money growth rate by comparing the results listed in penal 1, 2 and 3 of Table 4. At the default level $v = 0.9968$, the welfare cost of default is 0.0244 percentage of real output when expected inflation is low ($g = 0.9965$). When growth rate of money supply is higher ($g = 1.0149$), the welfare cost correspondingly is increased to 0.0265 percent of real output. Thus, the presence of default causes higher welfare loss at the same level of repayment rate when money growth rate is higher. When money growth rate is higher, the nominal interest rate is higher at the same level of default rate.

It can be explained as follows. In "Fisher Effect" (Proposition 1), the change in inflation is greater than the change of real interest rate. Combining with the facts that more money supply pushes up price level and corporate loan is relatively constant, the level of consumption decreases ("On-the-Verge Conditions", Proposition 3). Accordingly, with higher marginal utility of consumption, nominal interest rate in this economy increases, which further press down consumption stream.

In section 6.1, cyclical properties show that at the same level of credit-regulation, when money supply increases, the default rate will decreases. It explains why central bank injected money into the economy during the financial crisis. But as discussed above, those policies lead the economy into financially more fragile regime which results in the greater loss of social welfare with respect to the same level of default.

In this economy, markets are complete so each agent can completely hedge the risk. When firms are allowed to default, they face default penalties, causing social welfare loss. Detailed discussion abut social welfare analysis of default can be found in Dubey et al. (2000). According to their discussion, there are four drawbacks of the presence of default. First, creditors are less likely to lend when they rationally anticipating that they might not be paid. Second, borrowers may default on their promises regardless of their ability to repay and contingencies that have been foreseen. Third, imposing default penalty is a welfare loss as it does not transfer social resources. Fourth, those agents who default impose an externality on those reliable agents who do not default. Due to information asymmetric, those agents who will fulfill their obligations cannot distinguish themselves from those who are not reliable, and so reliable agents have to obtain the less favorable financing cost. Such externality problem can be modified by imposing penalties on defaulters (Akerlof, 1970).

But one thing needs to clarify here is that when the market is incomplete, it's a totally different story. According to Dubey et al. (2000), the social benefit of default option is likely to outweigh social cost of default in incomplete market. To begin with, an individual agent who decided to default at an optimal rate actually changes the given asset to a new one. Such a new asset is closer to his individual needs according to the benefit and cost caused by default. Moreover, it is a more desirable outcome in the perspective of the whole society. If individuals are not allowed to default, they only have the same given asset (fully repay their debts). But if there's default option, each individual agent can change the given asset to a favorable on his idiosyncratic need. He/She can choose any repayment rate from 0 to 1. Each agent can replace the given asset by a new one, thus one given asset is extended to many. With a larger asset span, the social welfare is improved.

6.2.2 Welfare cost of default: short run effect

In order to estimate the short run welfare cost of default, we follow Lucas (1987) to calculate the welfare cost of default as the gain from eliminating default. It is estimated as the minimum percentage increase in the level of consumption in every period needed to render the agent indifferent between the economies with and without default.

We do 100 times simulations with 141 periods and take the mean as the welfare measures. The burn-up period is 100.

⟨Insert table 5 here⟩

As in the steady state, default causes social welfare loss. However, under the same level of default regulation the sensitivity of default cost relative to the money growth rate is not significant. Thus, in the short run it may illude people that unconventional expansionary monetary policies such as QE and Abenomics will not bring deterioration in terms of welfare cost of default. But as we showed in the long run effect, if these expansionary monetary policies last long, it actually leads to higher social welfare loss. The short run and long run effects further confirm the propagation effect of the firm's optimal repayment behaviour.

7 Concluding remarks

In sum, this paper employs an extended monetary "dynamic general equilibrium model" (DSGE) with CIA constraints to analyze the impact of money and default on real economic activities, especially default. It is the first attempt to clarify the role of endogenous discrete default in infinite horizon CIA economy where positive response of output to monetary shock holds. In addition, this study further estimates the welfare cost of default under various conditions based on this extended RBC model with CIA constraints and default. Finally, this paper studies the long-run effects of default on real business cycles by revealing steady-state implications of this artificial economy.

Before welfare analysis, this study constructs non-pecuniary and non-linear default penalty with certain parameters in a monetary DSGE model with CIA constraints to overcome the limitation of standard CIA models in generating positive relationship between output response and monetary shock. All the analysis in this paper is under the framework of the extended monetary DSGE model.

According to the quantitative analysis, endogenous default not only has significant influence on the short-run economic fluctuations but also has long-run effects on the real economic activities. When default is considered as an equilibrium phenomena, the presence of default alters the transmission mechanism of technology and monetary shock through credit market. The interest rate closely link the credit market with the real economic activities. On the one hand, the interest rate reflects the default risk premium in the credit market. When default rate is high, the interest rate is high due to high default risk premium ("Fisher Effect", Proposition 1). And also, the interest rate itself will influence the default behavior in this economy. On the other hand, the interest rate plays important role in real economic activities because it reflects the financing cost of investment activities. When interest rate is high, firms hesitate more to borrow money. Thus, in the economy with default, it generates financial accelerator effect due to interaction between financial frictions and real economic activities. The presence of endogenous default amplifies and propagates the effects of shocks to real economy via credit market.

According to the welfare analysis, the default option induces social welfare loss. This study checks the welfare costs of default under different conditions and reveals that the amount of welfare cost induced by default varies with the levels of default rate and money growth rate. First, the welfare cost of default increases with the severity of default. When default rate is higher, the welfare cost caused by default is higher. Second, the welfare cost of default increases with the money growth rate at a given level of default. In high inflation economy, given a certain repayment rate, the welfare cost of default is high. While in low inflation economy, the welfare cost is low. Third, though the welfare cost of default is high in the long run, it is inconspicuous in the short run.

In the end, it is important to note the methodological limitations of the studies involved in this thesis. First, most of parameters in this model are calibrated based on standard literature and simple statistics of the U.S. historical data. The future research can use Bayesian Estimation to adjust parameters based on the performance of the target economy. Second, to construct the monetary policy rule, it assumes a simple case where money growth rate in the steady state never changes, while in reality there are shifts in the steady state of money growth rate corresponding to economic conditions. It's more reasonable to investigate more specific monetary policy rules such as the regime switching model. [†] Finally, this study models the cost of default as a reputation loss. With certain specific parameters of default penalty, it can successfully generate hump-shaped positive response of output to positive monetary shock. But the generalized methods to introduce default penalty is to be discussed in the future study.

[†]It would be perfectly possible to expand this exercise to include feedback rules in monetary policy, i.e. the nominal interest rate stipulated by the central bank via the money supply would positively respond to the inflation and output gap. However, in this case, the growth rate of the money supply as monetary policy is modeled by an exogenous process whose log-linearized forms follow an AR(1) process, for examining the effects of transitory monetary policy shocks.

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Appendices

Appendix A. List of detrended equations

Since money has stochastic trends and grows at a constant rate in the long run, the artificial economy is non-stationary. To begin with, all real variables $q_t = [Y_t, C_t, I_t, K_{t+1}]$ and the labor N_t are stationary because there is no technology or population growth. However, all nominal variables $Q_t = [L_t, W_t, P_t]$ grow with the money supply M_t. Thus, all nominal variables in the economy should be detrended to obtain a steady state. In detail, all the nominal variables should be divided by the money supply in this economy, i.e. $\hat{Q}_t = Q_t/M_t$ (Griffoli, 2007). The variables with hats represent stationary variables after detrending. Finally, the set of detrended equations of this model economy is listed as follows,

$$\begin{aligned}
\ln A_t &= \rho_A \ln A_{t-1} + (1-\rho_A)\ln \bar{A} + \sigma_A \varepsilon_{A,t} \\
\ln g_t &= \rho_g \ln g_{t-1} + (1-\rho_g)\ln \bar{g} + \sigma_g \varepsilon_{g,t} \\
E_t\left(\frac{\hat{P}_t}{C_{t+1}\hat{P}_{t+1}g_t}\right) &= E_t\left(\frac{\beta\hat{P}_{t+1}}{C_{t+2}\hat{P}_{t+2}g_{t+1}}\left(\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)\right)\right) \\
\hat{W}_t &= \frac{\hat{L}_t}{N_t} \\
\frac{\hat{L}_t}{N_t} &= \frac{\phi C_t \hat{P}_t}{(1-\phi)(1-N_t)} \\
\frac{(1-\alpha)\hat{P}_t Y_t}{\hat{L}_t} &= R_t \\
\frac{1}{R_t} &= \beta E_t\left(\frac{\nu_t C_t \hat{P}_t}{C_{t+1}\hat{P}_{t+1}g_t}\right) \\
C_t + K_{t+1} &= (1-\delta)K_t + K_t^{\alpha}(A_t N_t)^{1-\alpha} \\
\hat{P}_t C_t &= g_t \\
Y_t &= K_t^{\alpha}(A_t N_t)^{1-\alpha} \\
E_t\left(\frac{1}{C_{t+1}\hat{P}_{t+1}g_t}\right) &= \iota\left[(1-\nu_t)R_t\hat{L}_t\right]^{\eta}
\end{aligned}$$

Appendix B. Basic CIA Economy without default

This part describes an basic CIA economy without default, which is the starting point of the analysis in this paper. Default is not allowed in this economy and firms will pay back all their debts to households. There is no agency problem in this economy.

Optimization problem of households

Households solve the optimization problem as follows,

$$\max_{\{C_t,H_t,m_{t+1},D_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \{(1-\phi)\ln C_t + \phi \ln(1-H_t)\}$$

$$s.t.$$

$$P_t C_t \leq m_t - D_t + W_t H_t + (g_t - 1) M_t \tag{16}$$

$$0 \leq D_t \tag{17}$$

$$m_{t+1} = (m_t - D_t + W_t H_t + (g_t - 1) M_t - P_t C_t) + R_t D_t + F_t \tag{18}$$

Optimization problem of firms

Firms in this economy face the optimization problem as follows,

$$\max_{\{F_t,K_{t+1},N_t,L_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left\{ \frac{F_t}{C_{t+1} P_{t+1}} \right\}$$

$$s.t.$$

$$W_t N_t \leq L_t \tag{19}$$

$$F_t = L_t + P_t \left[K_t^{\alpha} (A_t N_t)^{1-\alpha} - K_{t+1} + (1-\delta) K_t \right] - W_t N_t - R_t L_t \tag{20}$$

Appendix C. AR(1) process estimation

In this paper, external technology and monetary shocks are constructed as stochastic AR(1) process. The AR(1) parameters are estimated from the time series data of the calculated labor-augmenting technology factor and the money stock M1 for the U.S.

Money growth rate: In this model, the monetary policy follows stochastic AR(1) process,

$$\begin{aligned} \ln g_t &= \rho_g \ln g_{t-1} + (1-\rho_g)\ln \bar{g} + \sigma_g \varepsilon_{g,t} \\ \varepsilon_{g,t} &\sim i.i.d.\ N(0,1) \end{aligned} \tag{21}$$

where ρ_g represents smoothing coefficient of money supply growth rate, $\varepsilon_{g,t}$ captures the unexpected monetary shock, and σ_g is the variance of the random shock.

To estimate AR(1) coefficient ρ_g and innovation variance σ_g , the money supply growth rate is calculated by divide the current period M1 by last period M1. Then take logarithm for the M1 growth rate and run 1 lag auto-regression for log form of M1 growth rate.

According to the results of the AR(1) regression, the smoothing coefficient of money supply growth rate is $\rho_g = 0.3522$ and the variance of monetary shock is $\sigma_g = 0.0168$. The unconditional mean of money growth rate is $\bar{g} = 1.0147$.

Technology evolution process: In this model, the monetary policy follows stochastic AR(1) process,

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + (1-\rho_A)\ln \bar{A} + \sigma_A \varepsilon_{A,t} \\ \varepsilon_{A,t} &\sim i.i.d.\ N(0,1) \end{aligned} \tag{22}$$

where ρ_A represents smoothing coefficient of labor-augmenting technology factor, $\varepsilon_{A,t}$ captures the unexpected technology shock, and σ_A is the variance of technology innovation.

To estimate AR(1) coefficient ρ_A and innovation variance σ_A , the labor-augmenting factor data is derived from the "Cob supply growth rate is calculated by the "Cobb-Douglas production" function, $Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}$, as follows,

$$\ln(A_t) = \frac{\ln(Y_t) - \alpha \ln(K_t)}{1-\alpha} - \ln(N_t)$$

According to the results of the AR(1) regression, the smoothing coefficient of money supply growth rate is $\rho_A = 0.8063$ and the variance of monetary shock is $\sigma_A = 0.0066$. The stationary technology is standardized to 1.

Appendix D. Figures and Tables

Figures

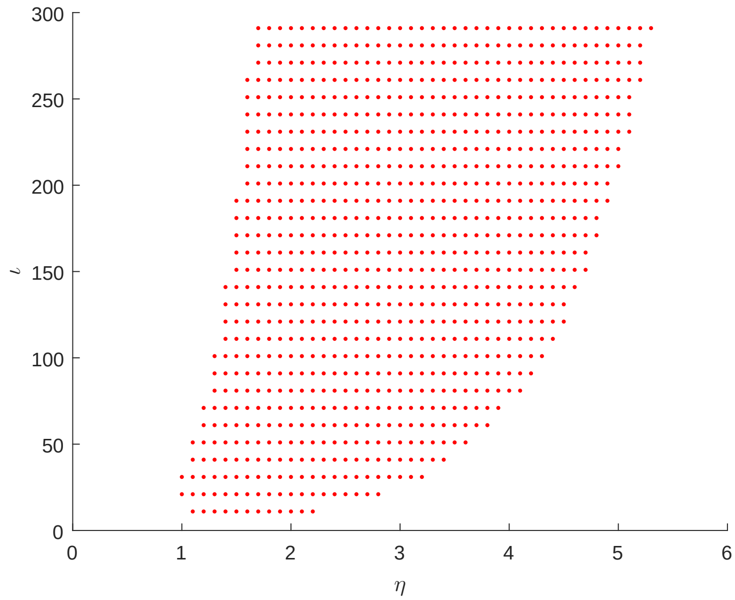

Figure 2: Parameter zone

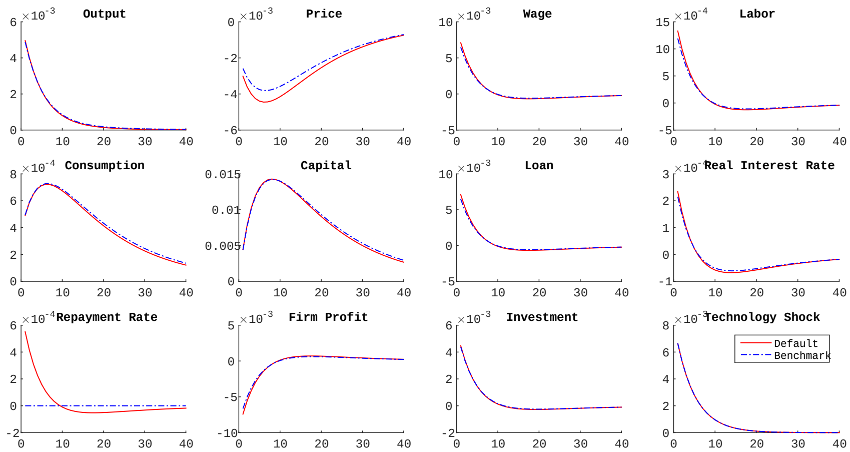

Figure 3: IRFs of positive productivity shock

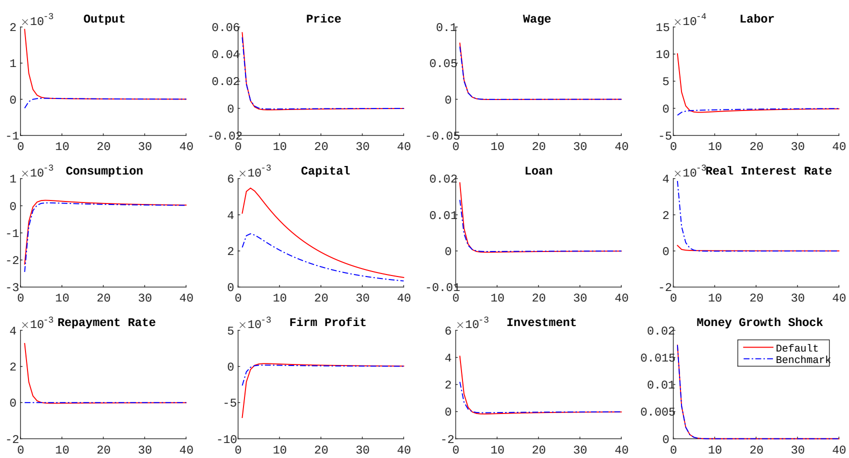

Figure 4: IRFs of positive money growth shock

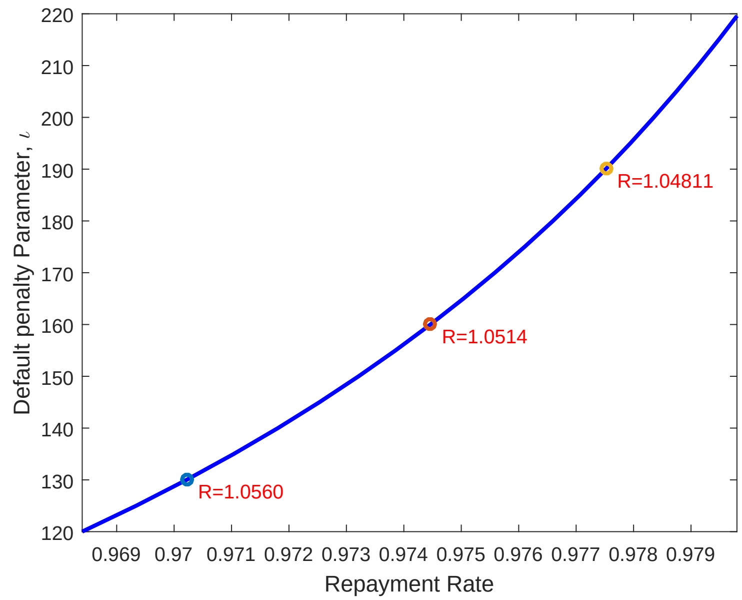

Figure 5: Sensitivity analysis of $\bar{\iota}$ and $\bar{\nu}$

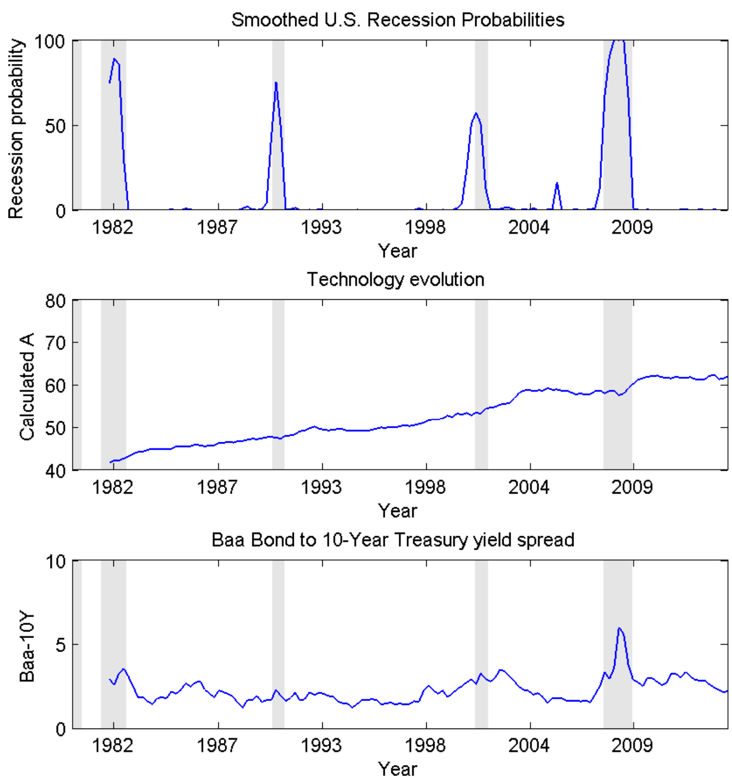

Figure 6: Recession probability, technology and credit risk

Tables

Table 1: Implied parameters

Description	Parameter	Value
Output elasticity of capital	α	0.3200
Discount factor	β	0.9900
Consumption and leisure preference shifter	ϕ	0.7730
Depreciation rate	δ	0.0250
Coefficient of default penalty	ι	150.00
Elasticity of non-pecuniary default	η	2.0000
Steady state labor-augmenting technology factor	$\bar{A}$	1.0000
Steady state money supply growth	$\bar{g}$	1.0147
AR(1) coefficient of technology	ρ_A	0.8063
AR(1) coefficient of money supply growth	ρ_g	0.3522
Standard deviation of technology innovation	σ_A	0.0066
Standard deviation of monetary policy innovation	σ_g	0.0168

Table 2: Statistics of U.S. and Artificial Economics

		Standard Deviation (Percentage)	Correlation with Output
U.S. time series	Output	0.0168	1.0000
	Consumption	0.0091	0.8528
	Investment	0.0597	0.9109
	Capital Stock	0.0061	0.4195
	Hours	0.0177	0.8489
	Productivity	0.0112	0.0351
	CPI	0.0069	0.2608
	GNP Deflator	0.0046	0.2576
Economy with default	Output	0.0059 (0.0006)	1.0000 (0.0000)
	Consumption	0.0035 (0.0003)	0.2884 (0.1158)
	Investment	0.0059 (0.0006)	0.8216 (0.0381)
	Capital Stock	0.0164 (0.0030)	0.3219 (0.0757)
	Hours	0.0015 (0.0002)	0.9778 (0.0055)
	Productivity	0.0080 (0.0008)	0.9926 (0.0018)
	Price Level	0.5251 (0.4500)	-0.0676 (0.1601)
Economy without default	Output	0.0059 (0.0006)	1.0000 (0.0000)
	Consumption	0.0034 (0.0003)	0.1708 (0.1267)
	Investment	0.0063 (0.0006)	0.8431 (0.0374)
	Capital Stock	0.0170 (0.0030)	0.3121 (0.0831)
	Hours	0.0015 (0.0002)	0.9838 (0.0046)
	Productivity	0.0079 (0.0008)	0.9997 (0.0001)
	Price Level	0.5115 (0.4638)	-0.0552 (0.1614)

Table 3: Statistics of historical yield spreads and M1 growth rate

	Baa-10Y spread	Aaa-10Y spread	M1 growth rate
Average	2.3511%	1.2881%	1.0149
Standard deviation	0.7177%	0.5035%	0.0184
Minimum	1.3700%	0.0100%	0.9814
Maximum	5.5900%	2.5600%	1.0986

Note:

1. Baa-10Y spread: Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity, Percent, the US data, 1981Q4-2016Q4

2. Aaa-10Y spread: Moody's Seasoned Aaa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity, Percent, the US data, 1981Q4-2016Q4

3. Growth rate of the US M1, 1981Q4-2016Q4.

Table 4: Steady states and welfare costs

			Quarterly Repayment Rate			
Quarterly Constraint		ν	1.0000	0.9968	0.9941	0.9906
Steady State		ι		2905.1	1247.0	640.58
$g = 0.9965$	Output		0.5774	0.5760	0.5747	0.5730
	Price		2.2400	2.2455	2.2504	2.2571
	Wage		4.2724	4.2697	4.2674	4.2642
	Labor		0.2041	0.2036	0.2031	0.2025
	Consumption		0.4458	0.4447	0.4438	0.4424
	Capital Stock		5.2640	5.2510	5.2397	5.2241
	Corporate Loan		0.8719	0.8692	0.8669	0.8636
	Nominal Interest Rate		1.0087	1.0118	1.0146	1.0184
	Real Interest Rate		1.0101	1.0132	1.0160	1.0198
	Firm Profit		0.1191	0.1218	0.1242	0.1275
Welfare Costs	$\triangle C/C \times 100$		0.0000	0.0316	0.0597	0.0989
	$\triangle C/Y \times 100$		0.0000	0.0244	0.0461	0.0763
Steady State		ι		2755.4	1182.7	607.56
$g = 1.0149$	Output		0.5702	0.5688	0.5675	0.5659
	Price		2.3045	2.3102	2.3152	2.3221
	Wage		4.3266	4.3239	4.3215	4.3183
	Labor		0.2015	0.2010	0.2006	0.2000
	Consumption		0.4402	0.4391	0.4382	0.4369
	Capital Stock		5.1981	5.1853	5.1740	5.1586
	Corporate Loan		0.8719	0.8692	0.8669	0.8636
	Nominal Interest Rate		1.0248	1.0279	1.0307	1.0346
	Real Interest Rate		1.0101	1.0132	1.0160	1.0198
	Firm Profit		0.1210	0.1238	0.1262	0.1295
Welfare Costs	$\triangle C/C \times 100$		0.0000	0.0343	0.0644	0.1070
	$\triangle C/Y \times 100$		0.0000	0.0265	0.0497	0.0826

Table 4: Steady states and welfare costs

Quarterly Constraint		Quarterly Repayment Rate ν	1.0000	0.9968	0.9941	0.9906
Steady State		ι		2615.5	1122.7	576.72
$g = 1.0333$	Output		0.5631	0.5617	0.5605	0.5588
	Price		2.3699	2.3758	2.3809	2.3881
	Wage		4.3807	4.3780	4.3757	4.3725
	Labor		0.1990	0.1985	0.1981	0.1975
	Consumption		0.4348	0.4337	0.4328	0.4315
	Capital Stock		5.1339	5.1211	5.1100	5.0948
	Corporate Loan		0.8719	0.8692	0.8669	0.8636
	Nominal Interest Rate		1.0408	1.0440	1.0469	1.0508
	Real Interest Rate		1.0101	1.0132	1.0160	1.0198
	Firm Profit		0.1229	0.1257	0.1282	0.1315
Welfare Costs	$\triangle C/C \times 100$		0.0000	0.0370	0.0692	0.1150
	$\triangle C/Y \times 100$		0.0000	0.0286	0.0534	0.0888

Note:

1. This paper measures the corporate bond yield spread as the difference between corporate bond and 10-year treasury bond. According to "Fisher Effect" (Proposition 1), quarterly repayment rate ν approximately equals to $e^{-YieldSpread/4}$. For the case of low default rate $\nu = 0.9968$, we use Moody's seasoned Aaa corporate bond yield relative to yield on 10-year treasury constant maturity. For the case of $\nu = 0.9941$, we use Moody's seasoned Baa corporate bond yield relative to yield on 10-year treasury constant maturity. For the case of high default rate $\nu = 0.9906$, we use $\mu_{Baa-10Y} + 2\sigma_{Baa-10Y}$.

2. To imitate the conditions of low, moderate and high inflation tax, we use quarterly historical U.S. M1 growth rate data from 1981Q4 to 2016Q4 with mean $\mu_g = 1.0149$, and standard deviation $Std_g = 0.0184$. The growth rate of money supply for each case is $(\mu_g - \sigma_g)$, (μ_g), and $(\mu_g - \sigma_g)$.

3. Interest rate adjusted for inflation, i.e. Real interest rate=Nominal interest rate/g.

Table 5: Welfare cost of default in dynamic

$\eta = 2$	g	$\Delta C/C$	ν
$\iota = 120$	0.9965	0.0150	0.8956
	1.0149	0.0150	0.8958
	1.0333	0.0150	0.8959
$\iota = 200$	0.9965	0.0148	0.8966
	1.0149	0.0148	0.8967
	1.0333	0.0148	0.8968
$\iota = 245$	0.9965	0.0147	0.8969
	1.0149	0.0148	0.8970
	1.0333	0.0148	0.8971